Taxable Income on the Job Ladder:

On-the-Job Search and the Welfare Cost of Labor Taxation

Brendan Epstein* Musa Orak† Elena Patel‡

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Abstract

The elasticity of taxable income (ETI) is often used to summarize the efficiency cost of taxation, but earlier work shows that search frictions can create a substantial gap between the ETI prediction and actual deadweight loss (DWL). We ask which features of a job ladder change this gap. In a calibrated search-and-matching model, we introduce these features sequentially. Duplicating the recruitment market leaves the gap unchanged, as does a controlled wage hierarchy. Allowing workers to climb the ladder through on-the-job search, on the other hand, increases the gap by 19 percent. The magnitude of this effect is not identified by standard labor-market observations and the ETI alone. Across calibrations that match the same targets but differ in unobserved nonwage amenities, on-the-job search increases the gap by up to 41 percent. An observation on the taxable wage bill as a share of output can distinguish these calibrations. Once workers move along a job ladder, taxable income alone need not contain enough information to measure the welfare cost of taxation.

JEL classification: H21, J64, J63, H24, E24.

Keywords: Elasticity of taxable income; deadweight loss; labor taxation; on-the-job search; job ladders; search externalities.

*Corresponding author. Department of Economics University of Massachusetts Lowell, Health and Social Sciences Building, Room 347, 113 Wilder St. Lowell, MA 01854. E-mail: epsteinb@gmail.com.

†Division of International Finance, Board of Governors of the Federal Reserve System, Washington, D.C. 20551, United States. E-mail: musa.orak@frb.gov. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. All errors are our own.

‡The Brookings Institution, 1775 Massachusetts Ave NW, Washington, DC 20036. E-mail: epatel@brookings.edu.

1 Introduction

The elasticity of taxable income (ETI) is a central parameter in tax policy analysis because, under suitable conditions, it provides sufficient information to measure the welfare cost of taxation, or deadweight loss (DWL) (Feldstein, 1999; Chetty, 2009a; Saez et al., 2012; Chetty, 2009b). The response of taxable income to the net-of-tax rate summarizes a wide variety of behavioral adjustments, including changes in hours, effort, reporting, and the mix of taxable and untaxed compensation, so these adjustments need not be observed directly. In competitive and neoclassical benchmarks, the ETI measures DWL exactly, including when workers and firms respond to taxes by shifting pay toward untaxed job amenities. Search and matching, however, break this equivalence. Epstein et al. (2023) show that the ETI can then substantially understate DWL, even with endogenous untaxed amenities. Recruiting uses real resources, and existing matches have value to firms that taxable income does not record.

This paper asks whether a job ladder changes the gap between actual and ETI-based DWL (the ETI–DWL gap) and, if so, which features of the ladder account for the change. The question matters because direct job-to-job mobility is a prominent feature of the U.S. labor market. Using updated Current Population Survey (CPS) measures, Fujita et al. (2024) document direct employer-to-employer transition rates of roughly 2–3 percent of employed workers per month. Workers also tend to move from lower- to higher-paying firms, and the firm-wage ladder is highly procyclical, even collapsing during the Great Recession (Haltiwanger et al., 2018). Thus the labor market contains both wage differences across jobs and substantial movement among them. If that movement changes the welfare information contained in taxable income, then the same estimated ETI may imply different welfare costs in labor markets that differ in their degree of mobility, including the same labor market at different points in the business cycle.

We reach two main conclusions. First, among the defining features of a job ladder, only direct job-to-job mobility changes the ETI–DWL gap in our controlled comparisons. Second, standard labor-market observations and the ETI alone do not identify the magnitude of

the change. Establishing the first conclusion requires separating elements that typically appear together: firms recruit in multiple markets, jobs differ in pay and value, and employed workers can move directly from one job to another. Distinguishing among these channels matters both for the economic mechanism and for the information needed to infer welfare from observed taxable-income responses.

We therefore introduce these channels sequentially in a general equilibrium search-and-matching model and quantify their effects in a calibrated economy. Starting from a one-job environment, we first duplicate the recruitment market without otherwise creating economically different jobs. We then introduce a wage hierarchy, in which the upper of two job types pays a higher real wage, while continuing to rule out direct movement between them. In the final step, lower-rung employees can reach upper-rung jobs through on-the-job search (OTJ), and observed worker flows discipline this mobility. Throughout the sequence, compensation includes both taxable real wages and untaxed job amenities, as in the one-job benchmark of Epstein et al. (2023).

Table 1 summarizes our research design and headline results. Market multiplicity by itself does not change the ETI–DWL gap. When the single recruitment market is duplicated into two otherwise identical markets, workers and vacancies are simply divided across two markets with identical technologies and compensation, so the aggregate allocation and its response to taxes are unchanged. The ETI and actual DWL therefore move exactly as they do in the one-job environment developed in Epstein et al. (2023). This first result establishes that the mere existence of multiple recruitment markets is not what changes the welfare information contained in taxable income.

A wage hierarchy also leaves the gap unchanged in our controlled benchmark. To isolate wage heterogeneity, we introduce a real-wage premium for upper-rung jobs while preserving the common aggregate targets and requiring the same ratio of vacancies to searchers in the two recruitment markets. The higher wage is supported by corresponding differences in productivity, recruiting costs, and match values, so the two rungs are economically distinct.

Table 1: Research design and headline results

Specification	**Recruitment markets**	**Wage hierarchy**	**On-the-job search**	**Effect on ETI–DWL gap**
One-job starting point	One	No	No	Baseline gap
Duplicated recruitment market	Two	No	No	Unchanged
Wage hierarchy	Two	Yes	No	Unchanged
Mobile job ladder	Two	Yes	Yes	Increases by 19%
Alternative admissible mobile-ladder calibrations	Two	Yes	Yes	Increases by up to 41%

Notes: The first four rows introduce the defining features of the job ladder sequentially. The final row varies unobserved compensation and resource allocations while preserving the maintained labor-market observations. The 41% figure is the largest paired increase from on-the-job search within the reported admissible range.

Nevertheless, these differences redistribute wages and match values across job types without changing aggregate employment, taxable income, recruiting expenditures, profits, or their responses to taxation. The invariance of the ETI–DWL gap is therefore an analytical result rather than a numerical coincidence. This result is specific to our controlled experiment; it does not imply that arbitrary wage dispersion is irrelevant for welfare.

Mobility through the hierarchy changes the result. Once lower-rung workers can search for and move directly to upper-rung jobs, the ETI–DWL gap increases by 19 percent, from 7.2 to 8.6 percent of predicted DWL. The intuition is that recruiting a nonemployed worker creates a new employment relationship, whereas recruiting an already-employed worker moves a worker from one existing employment relationship to another and destroys the match left behind. As a result, job-to-job mobility changes recruiting-resource use and the values of the employment relationships being created and left behind in ways that are not fully recorded by the response of taxable income. Mobility also shifts employment toward upper-rung jobs, in which firms' match values are higher, raising profits relative to the taxable wage bill without changing the ETI. The ETI consequently omits more of the welfare adjustment once workers can climb the ladder through on-the-job search.

This increase is captured by the sequential, target-matched comparison, in which intro-

ducing mobility also requires recalibrating latent model parameters to preserve the common observed targets. Because the recalibration could itself affect the gap, we conduct a complementary experiment that starts from the calibrated mobile economy and shuts off only employed workers' access to upper-rung jobs, holding all other model primitives fixed. Restoring on-the-job search in this experiment increases the ETI–DWL gap by 18 percent. That the two experiments yield similar results indicates that the quantitative importance of mobility is not an artifact of the recalibration used to construct the sequential comparison.

The size of the mobility effect, however, is not pinned down by the labor-market observations used in our baseline calibration. The baseline assumes that the two job rungs provide equal per-worker nonwage amenities, which is a calibration restriction rather than an observed feature of the labor market. We therefore vary the relative amount of nonwage amenities provided by the two rungs and, for each value, recalibrate the unobserved model primitives so that the economy continues to reproduce the same aggregate labor-market targets, real-wage premium, mobility moments, and aggregate amenity expenditure used in the baseline. Because these maintained targets imply the same ETI in our model, the alternative calibrations also generate the same ETI.

The resulting economies can look the same along all of the dimensions we require them to match while differing in their underlying resource allocation, including in the amount of output paid as taxable wages and in the recruiting costs, profits, and nonwage compensation associated with the two types of jobs. These differences matter for welfare because the ETI measures the response of taxable income, not the full resource consequences of employment and recruiting. Quantitatively, this identification problem is important. Across the admissible target-preserving latent calibrations, the increase in the ETI–DWL gap attributable to on-the-job search can reach 41 percent of the corresponding no-on-the-job-search gap. Thus, economies with the same ETI, the same aggregate labor-market outcomes, and the same worker flows can differ substantially in the extent to which the ETI understates the welfare cost of taxation. This exercise is therefore about identification—what the maintained observations

do and do not pin down—rather than robustness to alternative parameter values.

Additional resource information can discipline this variation. In our model, the taxable wage bill relative to output varies across calibrations that reproduce the same maintained labor-market observations. A compatible observation on that ratio can therefore distinguish among the latent calibrations and sharpen the welfare inference. More broadly, once workers move through a job ladder, the ETI and standard labor-market flows need not contain enough information to infer the welfare cost of taxation. Information about how output is divided between taxable compensation and other employment-related resources can matter as well.

We make two contributions. First, we isolate which features of a job ladder change the welfare information contained in taxable income. By introducing the ladder sequentially, we show that neither the presence of multiple recruitment markets nor a controlled real-wage hierarchy changes the ETI–DWL gap. The gap widens only when workers can move directly through the hierarchy via on-the-job search. This decomposition matters because wage dispersion and job-to-job mobility are often observed together, making it easy to attribute their welfare consequences to the job ladder as a whole. Our results instead show that, in the controlled environment we study, the key distinction is not whether jobs pay differently but whether workers can move between them. Put differently, the welfare content of taxable income depends on how workers move through the labor market, not merely on the existence of wage differences across jobs.

Second, we show that standard labor-market observations need not identify the magnitude of the welfare effect created by that mobility. Economies that match the same aggregate labor-market targets, real-wage premium, worker-flow moments, and ETI can imply different ETI–DWL gaps because those observations do not fully reveal the underlying allocation of compensation and recruiting resources. We characterize this identification problem and show how additional resource information, such as the taxable wage bill relative to output, can distinguish among otherwise observationally similar calibrations. The implication is that once workers move through a job ladder, measuring their taxable-income response and observed

labor-market flows may not be sufficient for welfare analysis.

Our paper is related to three literatures. First, it contributes to work using sufficient statistics to evaluate the welfare effects of taxation and to work studying taxation in labor markets with search frictions. Chetty (2009b) develops the general sufficient-statistics approach to welfare analysis, while Kroft et al. (2020) derive sufficient-statistics tax formulas in an environment with unemployment and endogenous wage responses. Epstein et al. (2023) show that the ETI–DWL equivalence survives endogenous amenities in their competitive and neoclassical benchmarks but fails once labor search frictions are introduced. Related search-based tax papers characterize optimal taxation when participation and matching are frictional (Hungerbühler et al., 2006; Lehmann, Parmentier, and van der Linden, 2011). Our object is different. We take the tax system as given and ask whether observable taxable-income responses continue to contain the information needed for welfare analysis as the labor market acquires the features of a job ladder.

Second, our analysis builds on the literature on job ladders and on-the-job search. Classic models show how employed search and worker or firm heterogeneity generate wage dispersion and job-to-job mobility (Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002), and Moscarini and Postel-Vinay (2018) survey this literature through the job-ladder lens. Empirically, Haltiwanger et al. (2018) document pronounced ladder-like worker flows across firms ranked by wages, while Faberman et al. (2022) provide direct evidence on search by employed workers. Rather than asking primarily how wage dispersion or worker flows arise, we use the components of the job ladder as controlled experiments to determine which features change the welfare information contained in the ETI.

Two papers are especially close to our tax-welfare question. Epstein and Nunn (2013) study taxation, match quality, and nonwage compensation in a search environment with on-the-job mobility, but take contact probabilities as exogenous rather than modeling vacancy creation and market tightness. Bagger, Moen, and Vejlin (2021) study deadweight loss in a rich equilibrium job ladder and decompose distortions associated with worker search, job

ranking, and vacancy creation. Our analysis complements theirs by asking whether the ETI summarizes the resulting welfare loss, which features of the ladder cause that mapping to change, and what additional information is needed when it does. Related empirical work also shows that taxation can affect job turnover and the long-run allocation of labor (Gentry and Hubbard, 2004; Kreiner, Munch, and Whitta-Jacobsen, 2015).

Finally, the identification exercise connects to work on nonwage job value and compensating differentials. The idea that workers value job characteristics other than wages has a long tradition (Rosen, 1986), and nonwage characteristics play an explicit role in models of on-the-job search (Sullivan and To, 2014). Hall and Mueller (2018) provide modern evidence that nonwage job values are quantitatively important for search behavior. Our use of nonwage compensation is narrower: we ask whether latent differences in the resource composition of compensation can change tax-welfare inference while the observed labor-market targets and ETI remain fixed.

The remainder of the paper is organized as follows. Section 2 develops the job-ladder search-and-matching environment. Section 3 derives the relationship between the ETI and DWL and characterizes the welfare correction generated by search frictions. Section 4 describes the calibration strategy and the measurement of the job ladder. Section 5 presents the sequential experiments that move from identical recruitment markets to a wage hierarchy and then to worker mobility through on-the-job search. Section 6 studies compensation, identification, and sensitivity, including the target-preserving latent calibrations and the additional resource information that can distinguish among them. Section 7 concludes.

2 The Model

We extend the one-job economy in Epstein et al. (2023) to two recruitment markets with homogeneous workers and job types $i \in \{B, G\}$. The labels identify job types, not an intrinsic ranking; in the job-ladder economies, G is the destination rung when workers prefer moving

from B. Production is perfectly competitive, and a period is one month.

A representative household has unit mass. At the start of period t, $n_{B,t}$ and $n_{G,t}$ are the employed masses and s_t is the mass of nonemployed job searchers. Labor-force participation is

$$L_t = s_t + n_{B,t} + n_{G,t}.$$

The remaining $1 - L_t$ is outside the labor force.

A type-i worker produces y_i units of output, earns taxable real wage $w_{i,t}$, and receives $a_{i,t}$ units of untaxed amenities. Amenities cost one unit of final goods per unit of service, and a type-i vacancy costs γ_i units of final goods.

Nonemployed workers can find either job type: a fixed share $\alpha \in (0, 1)$ of their search exposure goes to the G market and $1 - \alpha$ to the B market. Workers employed in B also receive fixed effective exposure $o \geq 0$ to G vacancies. Exposure determines access, not costly or chosen search effort; a B worker moves to G with probability $\lambda_t \equiv o f_{G,t}$, where $f_{G,t}$ is the match probability per unit of G-market exposure. The two exposure parameters stay fixed under tax changes.

Jobs separate exogenously with probability ρ; two constant-returns matching functions generate new jobs. Each filled match bargains over taxable wages and untaxed amenities.

To isolate mobility rather than retention bargaining, compensation is renegotiated each period without commitment, retention offers, or counteroffers. A B worker leaves the incumbent match before bargaining with a destination G firm; nonemployment is therefore the disagreement state in the destination bargain. This spot-bargaining institution abstracts from strategic incumbent responses studied, for example, by Shimer (2006) and Gottfries (2026).

Labor income is taxed and revenue is rebated to the household, net of any unemployment benefits. The general dynamic economy permits risk-free household saving and corresponding firm borrowing; these positions drop out of the stationary benchmark. Appendix A gives the dynamic optimization problems.

2.1 The household

The household pools income and consumption across members. It chooses consumption, active nonemployed search, and bond holdings, with preferences

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + h(1 - L_t) + \sum_{i \in \{B,G\}} \zeta_i^a \log A_{i,t} \right],$$

where $\beta \in (0,1)$ is the discount factor, c_t is consumption, and $A_{i,t} \equiv a_{i,t} n_{i,t}$ is total amenity service from type i jobs. The increasing, concave functions u and h value consumption and nonparticipation; $\zeta_i^a > 0$ measures amenity tastes.

Define taxable labor income by

$$Z_t \equiv w_{B,t} n_{B,t} + w_{G,t} n_{G,t}.$$

The household budget is

$$c_t + \mathcal{B}_t = (1 - \tau_t) Z_t + \mathcal{I} s_t + R_{t-1}^b \mathcal{B}_{t-1} + T_t + \Pi_t,$$

where τ_t is the labor tax, $\mathcal{I}$ is the benefit per active nonemployed searcher, $\mathcal{B}_t$ denotes bonds, R_t^b their gross real return, T_t transfers, and Π_t household-owned firms' profits.

Employment at the start of t determines current production and compensation; matching in t determines employment in $t+1$. Let $f_{i,t}$ be the probability of a match per unit of exposure in market i, and define the B-to-G transition probability as

$$\lambda_t \equiv o f_{G,t}.$$

Job destruction and an upward move are mutually exclusive. New matches enter employ-

ment next period, without another separation draw. The laws of motion are

$$n_{B,t+1} = (1 - \rho - \lambda_t) n_{B,t} + (1 - \alpha) f_{B,t} s_t, \tag{2.1}$$

$$n_{G,t+1} = (1 - \rho) n_{G,t} + f_{G,t} \left(o n_{B,t} + \alpha s_t \right). \tag{2.2}$$

A hire from the G market can originate in either nonemployment or an existing B match; the latter destroys the incumbent match.

The household takes wages, amenities, matching probabilities, returns, profits, taxes, and transfers as given. Its dynamic optimization and the associated marginal employment values are derived in Appendix A.

2.2 Firms

Type-i firms produce y_i and pay $w_{i,t} + a_{i,t}$ per worker, while posting vacancies $v_{i,t}$ at unit cost γ_i.

Let $q_{i,t}$ be the vacancy-filling probability. With $R_{B,t} = 1 - \rho - \lambda_t$ and $R_{G,t} = 1 - \rho$, employment at each type of firm evolves as

$$n_{i,t+1} = R_{i,t} n_{i,t} + q_{i,t} v_{i,t}.$$

A B firm also loses workers through direct moves to G.

Firms are household-owned and may issue one-period risk-free debt. They choose vacancy creation and financing to maximize discounted dividends, taking compensation and matching probabilities as given. The full dividend and asset problems appear in Appendix A.

2.3 Matching and the job ladder

Let $S_{i,t}$ be total effective exposure to vacancies of type i. The fixed exposure shares imply

$$S_{B,t} = (1-\alpha)s_t, \tag{2.3}$$

$$S_{G,t} = on_{B,t} + \alpha s_t. \tag{2.4}$$

Each market has a constant-returns matching technology,

$$M_{i,t} = \phi v_{i,t}^{\xi} S_{i,t}^{1-\xi}, \qquad i \in \{B, G\}, \tag{2.5}$$

where $\phi > 0$ measures matching efficiency and $\xi \in (0,1)$ is the vacancy elasticity. With tightness $\theta_{i,t} \equiv v_{i,t}/S_{i,t}$, match and vacancy-filling probabilities are, respectively,

$$f_{i,t} \equiv \frac{M_{i,t}}{S_{i,t}} = \phi \theta_{i,t}^{\xi},$$

and

$$q_{i,t} \equiv \frac{M_{i,t}}{v_{i,t}} = \phi \theta_{i,t}^{\xi-1}.$$

The job-finding probability from nonemployment is

$$f_{U,t} = (1-\alpha) f_{B,t} + \alpha f_{G,t}.$$

For a B worker, the upward transition probability remains $\lambda_t = o f_{G,t}$.

Two markets alone do not constitute a job ladder: with identical jobs and $o = 0$ they simply divide recruitment. A ladder arises when G jobs provide strictly greater worker value and B employees can accept G contacts.

We restrict attention to feasible probabilities: $f_{i,t}, q_{i,t}, f_{U,t} \leq 1$ and $\rho + \lambda_t \leq 1$.

2.4 Compensation and bargaining

Each match renegotiates its current wage and amenities every period, without commitment to future compensation.

Let $\delta_{i,t}$ denote the household's surplus from a marginal member employed at type i relative to its optimized nonemployment alternative, and $J_{i,t}$ the firm's installed-match value. The worker's disagreement state is nonemployment and the firm's is an unfilled vacancy. For a B worker who moves to G, the old match is severed before the destination bargain.

A one-unit wage increase raises worker surplus by $(1-\tau_t)u'(c_t)$ and lowers firm value by one. An additional unit of amenities raises worker surplus by $\zeta_i^a/A_{i,t}$ and also costs the firm one unit of final goods.

The bargaining specification is conditional on our spot-contract institution. As emphasized by Shimer (2006), conventional surplus sharing need not apply when contracted wages affect turnover and hence total match surplus. We adopt the predetermined-search and repeated-renegotiation timing used by Krause and Lubik (2010), with the current-flow bargaining rationale also developed by Elsby and Gottfries (2022). In our environment, fixed on-the-job search exposure and strict upper-rung acceptance imply that a local current-wage deviation does not affect the lower-to-upper transition hazard. Together with the independence of future contracts from current compensation, this removes the relevant local wage–duration feedback (Appendix A.3). Gottfries (2026) provides a related foundation for linear surplus sharing in the continuous-renegotiation limit. Our bargaining conditions remain specific to this contracting institution and need not extend to endogenous search effort or alternative retention protocols.

For worker Nash weight $\psi \in (0,1)$, the current bargain solves

$$\max_{w_{i,t},a_{i,t}} \delta_{i,t}^{\psi} J_{i,t}^{1-\psi}.$$

The wage and amenity first-order conditions imply, with $b \equiv \psi/(1-\psi)$,

$$\delta_{i,t} = b(1-\tau_t)u'(c_t)J_{i,t},$$

and

$$A_{i,t} = \frac{\zeta_i^a}{(1-\tau_t)u'(c_t)}.$$

These conditions divide match surplus and determine aggregate amenity services. At equilibrium marginal valuations, a taxable wage and an amenity have equal bilateral value per unit of resource cost; the bilateral problem need not uniquely select their composition. The symmetric equilibrium conditions pin down the compensation allocation.

2.5 Government, market clearing, and equilibrium

The government rebates labor-tax revenue net of benefits:

$$T_t + \mathcal{I}s_t = \tau_t Z_t.$$

Output, amenity expenditure, and recruiting expenditure are

$$Y_t = y_B n_{B,t} + y_G n_{G,t}.$$

$$A_t \equiv A_{B,t} + A_{G,t} = a_{B,t}n_{B,t} + a_{G,t}n_{G,t},$$

$$K_t \equiv \gamma_B v_{B,t} + \gamma_G v_{G,t}.$$

Goods-market clearing requires

$$c_t + A_t + K_t = Y_t. \tag{2.6}$$

Household bonds are claims on firm debt, so financial flows cancel on aggregation. Appendix A states asset-market clearing explicitly.

An equilibrium consists of optimizing household and firm choices, the matching and employment transitions, Nash compensation, government budget balance, and goods and financial market clearing.

2.6 Benchmark specialization

For the tax-welfare comparison we follow the efficient benchmark of Epstein et al. (2023): no unemployment benefits ($\mathcal{I} = 0$) and quasilinear consumption utility ($u(c_t) = c_t$). This isolates job-ladder mechanisms from separate insurance distortions.

Marginal utility of nonparticipation is

$$h'(\ell_t) = \zeta \ell_t^{-\nu}, \qquad \ell_t \equiv 1 - L_t,$$

with $\zeta, \nu > 0$. The quantitative benchmark uses $\nu = 1$, equivalent to log utility in nonparticipation up to an additive constant.

Quasilinearity fixes the stochastic discount factor at β and makes asset positions irrelevant for the allocations; normalize household bonds and firm debt to zero. Employment and matching remain dynamic.

The resulting household employment-value and search equations are derived from the dynamic problem in Appendix A. We use their stationary forms below.

Similarly, the firm Bellman, free-entry, and bargaining conditions reduce to the stationary system below; their dynamic counterparts are in Appendix A.

The analysis compares stationary equilibria. For constant taxation define

$$t \equiv 1 - \tau, \qquad d_0 \equiv 1 - \beta(1 - \rho), \qquad d_B \equiv d_0 + \beta\lambda.$$

Stationary employment flows require

$$(\rho + \lambda) n_B = (1 - \alpha) f_B s, \tag{2.7}$$

$$\rho n_G = f_G(on_B + \alpha s). \tag{2.8}$$

Household search and employment values satisfy

$$h'(\ell) = \beta\left[(1-\alpha)f_B\delta_B + \alpha f_G\delta_G\right], \tag{2.9}$$

$$d_B\delta_B = tw_B + \frac{\zeta_B^a}{n_B} - h'(\ell) + \beta\lambda\delta_G, \tag{2.10}$$

$$d_0\delta_G = tw_G + \frac{\zeta_G^a}{n_G} - h'(\ell). \tag{2.11}$$

Installed-match values, free entry, and bargaining satisfy

$$d_BJ_B = y_B - w_B - a_B, \qquad d_0J_G = y_G - w_G - a_G, \tag{2.12}$$

$$\gamma_i = \beta q_i J_i. \tag{2.13}$$

$$\delta_i = tbJ_i, \qquad A_i = \frac{\zeta_i^a}{t}. \tag{2.14}$$

These equations, matching, and market clearing characterize stationary private equilibrium. With positive mobility we study equilibria in which G is strictly preferred by B workers; equal values are permitted in the identical-market case with $o = 0$.

With $\mathcal{I} = 0$, $T = \tau Z$ and $c = Y - A - K$. Aggregate firm profits obey the useful installed-match identity

$$\Pi = (1-\beta)\sum_{i\in\{B,G\}} J_i n_i. \tag{2.15}$$

3 Efficiency and the Tax-Welfare Relationship

The model separates three departures from the one-job economy in Epstein et al. (2023): a second market, differentiated jobs, and direct job-to-job movement. We first evaluate recruitment efficiency, then derive the stationary tax response and its welfare implications.

3.1 Efficiency and job-to-job mobility

The common planning problem

A single resource planner covers economies A (identical markets), B (differentiated jobs without mobility), C (the mobile ladder), and the fixed-primitives C off counterfactual. The one-market benchmark is a separate limit. The planner faces the same fixed exposure and matching technology as private agents; it cannot reassign workers or select the origin of a match.

The planner chooses nonemployed search s, vacancies v_B, v_G, and amenity services A_B, A_G, starting from inherited employment.

Suppress time subscripts and use primes for next period. Of total upper-market exposure S_G, the employed-origin share is

$$p \equiv \frac{on_B}{S_G} = \frac{on_B}{on_B + \alpha s}. \tag{3.1}$$

The remaining share $1 - p$ originates in nonemployment; when $o = 0$, $p = 0$.

The employment transitions are

$$n'_B = (1 - \rho)n_B + M_B - pM_G, \tag{3.2}$$

$$n'_G = (1 - \rho)n_G + M_G. \tag{3.3}$$

Each G match adds a G worker, but the fraction p drawn from B also eliminates an existing B match.

With value function $P(n_B, n_G)$, the resource planner solves

$$P(n_B, n_G) = \max_{s, \{v_i, A_i\}_{i \in \{B,G\}}} \Bigg\{ Y - A - K + h(1 - L) + \sum_{i \in \{B,G\}} \zeta_i^a \log A_i + \beta P(n'_B, n'_G) \Bigg\}, \tag{3.4}$$

subject to these transitions, matching, feasible participation, nonnegative consumption, and the probability bounds; $L = s + n_B + n_G$, $A = A_B + A_G$, and $K = \gamma_B v_B + \gamma_G v_G$. Transfers do not enter resource welfare.

Write the social continuation value of an additional type-i worker as

$$\mu_i \equiv \frac{\partial P(n'_B, n'_G)}{\partial n'_i}.$$

Appendix B gives the full planner derivatives and proofs.

Without on-the-job mobility

Without mobility ($o = 0$), every hire comes from nonemployment: recruitment creates a match without displacing another.

The planner's vacancy conditions are

$$\gamma_i = \beta \xi q_i \mu_i, \qquad i \in \{B, G\}, \tag{3.5}$$

where γ_i is vacancy cost and $\xi q_i \mu_i$ is the marginal social return from a new match.

Private free entry requires $\gamma_i = \beta q_i J_i$. At zero tax, $\mu_i = \delta_i + J_i$; under the Hosios restriction $\psi = 1 - \xi$ (Hosios, 1990), bargaining implies

$$J_i = \xi \mu_i, \qquad \delta_i = (1 - \xi)\mu_i.$$

Thus vacancy creation, household search, and amenity choice implement the planner allocation for both identical and differentiated jobs.

At positive taxes, private amenities satisfy $A_i = \zeta_i^a/(1 - \tau)$ rather than the resource planner's $A_i = \zeta_i^a$. To distinguish this tax distortion from recruitment inefficiency, restrict the planner to the private economy's participation and total amenity expenditure each period. The no-mobility allocation remains efficient conditional on these margins; the planner

allocates a given total amenity expenditure across jobs in the same proportions.

Thus the no-mobility benchmark establishes two distinct results. At zero tax, the decentralized allocation is efficient under the Hosios condition. At a positive tax, recruitment remains efficient conditional on the participation and amenity choices distorted by taxation. This distinction allows us to identify the additional inefficiency introduced by direct job-to-job movement rather than attribute existing tax distortions to the job ladder.

With on-the-job mobility

Now let B workers receive and accept G offers. A hire from nonemployment adds a new match, whereas hiring an employed worker replaces an existing B match. The respective social gains are μ_G and $\mu_G - \mu_B$.

With employed-origin share p, a G match has expected social value $\mu_G - p\mu_B$. Planner vacancy creation therefore satisfies

$$\gamma_B = \beta\xi q_B \mu_B, \tag{3.6}$$

$$\gamma_G = \beta\xi q_G(\mu_G - p\mu_B). \tag{3.7}$$

The term $p\mu_B$ is the expected value of the incumbent employment destroyed by G recruitment; the B vacancy condition is unchanged.

Private free entry instead uses $\gamma_G = \beta q_G J_G$, omitting the value of the displaced B match.

At zero tax, implementing both planner conditions requires type-specific surplus shares. The necessary worker bargaining weights are

$$\psi_B = 1 - \xi, \qquad \psi_G = 1 - \xi + \xi\frac{p\mu_B}{\mu_G}. \tag{3.8}$$

The upper-market worker weight exceeds $1 - \xi$ whenever $p\mu_B > 0$: a smaller firm share would temper vacancies to reflect displaced employment. A common bargaining weight cannot implement both planner conditions.

Proposition 1 (Efficiency and job-to-job mobility)**.** *Consider positive interior allocations under the fixed-exposure matching technology.*

(i) Without on-the-job mobility. If $o = 0$ and $\psi = 1 - \xi$, the zero-tax private allocation, starting from its equilibrium employment stocks, maximizes discounted resource welfare. At a positive tax, the no-OTJ private allocation maximizes resource welfare among feasible paths that reproduce its labor-force participation and aggregate amenity expenditure in every period.

(ii) With on-the-job mobility. If $o > 0$ and $p\mu_B > 0$, no common Nash bargaining weight implements an interior zero-tax planner allocation with both recruitment markets active.

Proof. See Appendix B. □

Mobility adds a displacement externality absent from duplication or wage heterogeneity: the destination firm does not account for an incumbent match that disappears.

Appendix B derives the full conditions, verifies the calibrated planner comparison, and constructs a feasible welfare-improving vacancy reduction. The quantitative benchmark keeps common Hosios bargaining power.

3.2 The equilibrium response to taxation

We next compare stationary equilibria as the tax rate varies, holding all parameters, including α and o, fixed.

Recruitment and compensation

Let $t = 1 - \tau$ and $b = \psi/(1 - \psi)$ as above. Define

$$Q \equiv \frac{h'(1 - L)}{t}, \qquad e_i \equiv w_i + a_i.$$

Here Q values nonparticipation in pretax goods units and e_i is the resource cost of compensation per worker.

Combining stationary worker and firm values, bargaining, search, and free entry yields

$$d_B(1+b)J_B = y_B - Q + \beta\lambda b J_G, \tag{3.9}$$

$$d_0(1+b)J_G = y_G - Q, \tag{3.10}$$

$$Q = \beta b\left[(1-\alpha)f_B J_B + \alpha f_G J_G\right], \tag{3.11}$$

$$J_i = \frac{\gamma_i}{\beta q_i}, \qquad i \in \{B, G\}. \tag{3.12}$$

The reduced system contains no explicit tax rate. Substituting matching probabilities and $\lambda = o f_G$ gives two equations for market tightness; where their Jacobian is nonsingular, tightness, probabilities, and match values are locally invariant to taxation.

Installed-match values also fix total compensation cost:

$$e_B = y_B - d_B J_B, \qquad e_G = y_G - d_0 J_G.$$

The tax therefore changes the split between wages and amenities, not recruitment or total compensation per worker.

Participation and taxable income

The stationary flows fix employment composition and the unemployment rate; employment, search, and vacancies instead scale with participation L.

Because Q is invariant, $h'(1-L) = tQ$. For $h'(\ell) = \zeta \ell^{-\nu}$ this gives

$$L_\tau = -\frac{1-L}{\nu t}, \tag{3.13}$$

where a τ subscript denotes differentiation with respect to the tax rate.

Thus a higher tax reduces participation and aggregate job-to-job moves, not the individual probability of climbing the ladder.

Bargaining implies $A_i = \zeta_i^a / t$, hence

$$A_\tau = \frac{A}{t}. \tag{3.14}$$

Untaxed amenities rise at the expense of wages within constant e_i.

Since $Z = \sum_i e_i n_i - A$, the taxable-income response is

$$Z_\tau = \frac{Z + A}{L} L_\tau - \frac{A}{t}. \tag{3.15}$$

The two terms capture lower participation and substitution from taxable wages toward untaxed amenities.

Define the ETI with respect to the net-of-tax rate by

$$\epsilon \equiv -\frac{t}{Z} Z_\tau. \tag{3.16}$$

Using the participation response gives

$$\epsilon = \underbrace{\left(1 + \frac{A}{Z}\right) \frac{1 - L}{\nu L}}_{\text{Participation channel}} + \underbrace{\frac{A}{Z}}_{\text{Amenity-substitution channel}}. \tag{3.17}$$

The first term measures lost compensation from lower participation, including its amenity costs; the second measures substitution from wages to untaxed amenities at a given employment allocation.

Proposition 2 (Equilibrium tax response and the ETI)**.** *Consider an interior equilibrium of the benchmark model with fixed search-exposure parameters and common Nash bargaining power. Suppose the Jacobian of the reduced two-market tightness system is nonsingular.*

Along the local equilibrium tax branch, labor-market tightness, job-finding and vacancy-filling probabilities, the climbing probability, firm match values, total compensation per worker, and Q remain constant. Employment shares and the unemployment rate remain constant,

while employment, search, and vacancies scale with labor-force participation.

For $h'(\ell) = \zeta \ell^{-\nu}$, the participation response is given by (3.13), and the elasticity of taxable income is the sum of the participation and amenity-substitution channels in (3.17).

Proof. See Appendix A. □

Hence economies sharing L, A/Z, and ν have the same ETI even if mobility changes their match composition and values.

3.3 Welfare and the ETI

To evaluate sufficiency, we compare the welfare derivative along the stationary tax branch with the ETI-based prediction.

The object is stationary flow welfare, not a transition-path welfare calculation.

The welfare and taxable-income derivatives

Under quasilinear utility, resource welfare is

$$W(\tau) = Y - A - K + h(1 - L) + \sum_{i \in \{B,G\}} \zeta_i^a \log A_i. \tag{3.18}$$

Taxes and lump-sum transfers cancel within the household.

Define actual marginal welfare loss in goods as $H \equiv -W_\tau$. The subscript denotes the total derivative along the equilibrium branch.

Differentiation using $\zeta_i^a / A_i = t$ gives

$$H = -Y_\tau + K_\tau + h'(1 - L)L_\tau + \tau A_\tau. \tag{3.19}$$

The output and recruiting terms measure net goods resources; the participation term values nonparticipation, and the amenity term accounts for the gap between amenity cost and marginal utility.

The ETI instead follows taxable wages. Firm accounting implies $Z = Y - A - K - \Pi$, where Π is aggregate profits. Define its marginal welfare prediction $H_\epsilon \equiv -\tau Z_\tau = (\tau Z/t)\epsilon$. The corresponding decomposition is

$$H_\epsilon = -\tau Y_\tau + \tau A_\tau + \tau K_\tau + \tau \Pi_\tau. \tag{3.20}$$

The amenity-response terms in H and H_ϵ cancel: amenities matter for the ETI, but under the maintained bargaining institution do not create a separate additional welfare term.

Normalize by the taxable wage bill: $D \equiv H/Z$ and $D_\epsilon \equiv H_\epsilon/Z = (\tau/t)\epsilon$. Subtracting the derivatives yields

$$\underbrace{D}_{\text{Deadweight loss}} = \underbrace{\frac{\tau}{t}\epsilon}_{\text{ETI prediction}} + \underbrace{\frac{-t(Y_\tau - K_\tau) + h'(1-L)L_\tau - \tau\Pi_\tau}{Z}}_{\text{Additional welfare channels}}. \tag{3.21}$$

The remaining channels capture changes in output net of recruiting, the utility value of nonparticipation, and the response of firm profits. Taxable wages alone do not account for every resource use or income claim.

The additional welfare correction

Proposition 2 implies that output, recruiting costs, and profits scale with participation:

$$Y_\tau = \frac{Y}{L}L_\tau, \qquad K_\tau = \frac{K}{L}L_\tau, \qquad \Pi_\tau = \frac{\Pi}{L}L_\tau. \tag{3.22}$$

Define the correction in goods units by $\Omega \equiv H - H_\epsilon$. Substitution into the welfare decomposition gives

$$\Omega = \left[h'(1-L) - \frac{t(Y-K) + \tau\Pi}{L}\right] L_\tau. \tag{3.23}$$

The worker-value, flow, and bargaining conditions imply the aggregate household identity

$$t(Z + A) - h'(1 - L)L = tb\Pi, \tag{3.24}$$

where L includes employed and active nonemployed workers.

Using $Y - K = Z + A + \Pi$ reduces the correction to

$$\Omega = -(1 + tb)\frac{\Pi}{L}L_\tau. \tag{3.25}$$

Equivalently, normalized welfare satisfies

$$D = \frac{\tau}{t}\epsilon - (1 + tb)\frac{\Pi}{Z}\frac{L_\tau}{L}. \tag{3.26}$$

Installed matches generate firm value that contracts with participation when taxes rise. The ETI records taxable-wage responses but not the full welfare cost of that contraction.

Positive match values imply positive profits by (2.15), and $L_\tau < 0$ by (3.13); hence $\Omega > 0$ and the ETI prediction understates DWL.

In the one-job economy of Epstein et al. (2023), the same forces appear as separate profit and search terms; here they combine into a single correction with the same structure in both economies, but mobility changes match composition and values, and therefore Π/Z and the magnitude of the correction.

Thus the ETI can be identical across economies with different welfare corrections: it depends on L, A/Z, and ν, while the correction also reflects installed-match values through profits.

Proposition 3 (The welfare correction)**.** *Consider an interior equilibrium on the local tax branch characterized in Proposition 2. Actual DWL in goods units equals the ETI-based*

prediction plus the correction in (3.25)*:*

$$H = H_\epsilon + \Omega, \qquad \Omega = -(1+tb)\frac{\Pi}{L}L_\tau. \tag{3.27}$$

Define the ratio of actual deadweight loss to the ETI prediction as $R \equiv D/D_\epsilon$*. For a positive interior tax rate, positive taxable income and match values, and finite positive* ν*,*

$$R - 1 = \frac{(1+tb)(\Pi/Z)(1-L)}{\nu L \tau \epsilon} > 0. \tag{3.28}$$

Proof. See Appendix A. □

The quantitative experiments apply these same welfare components to the sequential benchmark economies.

4 Calibration and the Measurement of a Job Ladder

We construct three economies that sequentially introduce a second recruitment market (A), differentiated jobs (B), and direct job-to-job mobility (C). Common aggregate targets isolate these features from changes in the aggregate reference economy. Additional survey moments discipline the wage hierarchy and mobility in C; the remaining restrictions are stated separately from empirical observations.

4.1 Common aggregate targets

For comparability with the one-job economy of Epstein et al. (2023), we retain its aggregate targets and parameter conventions rather than updating them with newer data. The monthly benchmark uses $\beta = 0.996$, labor tax rate $\tau = 0.189$, matching elasticity $\xi = 0.5$, and Nash worker weight $\psi = 0.5$ (the Hosios condition). Consumption is quasilinear, nonparticipation utility is logarithmic ($\nu = 1$), and unemployment insurance is zero.

The common targets are participation $L = 0.62$, vacancies per nonemployed searcher $V/s = 0.72$ (where $V = v_B + v_G$), amenity expenditure relative to taxable wages $A/Z = 0.11$, and output $Y = 1$. For unemployment $u = s/L$ and nonemployed job finding $f_U = (1-\alpha)f_B + \alpha f_G$, we use the model-consistent reconstruction of the inherited benchmark: matching efficiency $\phi = 0.507$ and separation $\rho = 0.027$ imply $f_U = 0.430204$ and $u = 0.059055$, evaluated at full precision. Appendix G reports the alternative that imposes the published rounded figures 0.43 and 0.058 exactly.[1] The inherited $A/Z = 0.11$ is retained for comparability, not presented as a newly verified measure of employers' amenity resource costs; evidence on nonwage job values, including Hall and Mueller (2018), does not directly measure this ratio.

Table 2: Targets and restrictions across the benchmark sequence

Object	A: identical	B: wage ladder	C: OTJ
Participation L	0.620000	0.620000	0.620000
Unemployment u	0.059055	0.059055	0.059055
Nonemployed finding f_U	0.430204	0.430204	0.430204
Vacancies per searcher V/s	0.720000	0.720000	0.720000
Output Y	1.000000	1.000000	1.000000
Amenity expenditure / wages A/Z	0.110000	0.110000	0.110000
Log wage premium m	0.000000	0.111702	0.111702
Climbs per employed r	–	–	0.003973
Nonemployed-origin share of upper hires η	–	–	0.451768
Per-worker amenity ratio a_G/a_B	1	1	1
Equal tightness $\theta_B = \theta_G$	Yes	Yes	Unrestricted
OTJ exposure o	0	0	.01

All three cases use $\alpha = 0.09605785$, $\beta = .996$, $\tau = .189$, $\xi = \psi = .5$ and $\nu = 1$. Common aggregate targets are evaluated at stored full precision. Climbing and the nonemployed-origin share of upper hires are targeted only in C. In A/B, $o = 0$ implies $r = 0$ and $\eta = 1$. Equal amenities and the exposure convention are calibration restrictions.

The log wage premium $m = \log(w_G/w_B)$ distinguishes B and C. Economy C also targets upward moves per employed worker r and the nonemployed-origin share of upper hires η, measured below. Equal baseline amenities, equal market tightness in A and B, and the scale

[1]The published rounded productivity 1.685 is not jointly consistent with $Y = 1$ and the reported participation and unemployment rates under linear one-type production. The reconstructed value is 1.714131 (1.712211 under the exact-rounded-target alternative), so published productivity is not imposed as an additional target.

of employed-worker exposure are *restrictions*, not further data. Neither ETI nor DWL is targeted: common L, A/Z, and ν imply a common ETI through (3.17), but the welfare correction is an equilibrium outcome.

4.2 Constructing the benchmark economies

All three economies share the aggregate targets, parameter conventions, and nonemployed upper-market exposure $\alpha = 0.09605785$, recovered from C and then held fixed in A and B. Their other free parameters are recalibrated as needed to retain the common targets.

Economy A: Two identical recruitment markets

With no mobility ($o = 0$), we split the one-job economy's employment N and vacancies V in proportion to exposure:

$$\begin{aligned} n_B &= (1-\alpha)N, & n_G &= \alpha N, \\ v_B &= (1-\alpha)V, & v_G &= \alpha V. \end{aligned} \tag{4.1}$$

Both jobs have the same productivity, vacancy cost, matching technology, and compensation; amenity tastes are split proportionally to preserve equal amenities per worker and aggregate expenditure. Constant returns to scale in matching preserves tightness and contact probabilities. The resulting change in logarithmic amenity utility is an additive, tax-independent constant, so neither the allocation nor its marginal tax-welfare response changes. There is no operative job ladder in A.

Economy B: Differentiated jobs without mobility

Economy B adds the survey-based premium $m = \log(w_G/w_B) = 0.111702$ while retaining $o = 0$ and equal amenities per worker, $a_B = a_G$. To isolate wage heterogeneity from differences

in recruitment conditions, we impose:

$$\theta_B = \theta_G = \frac{V}{s}. \tag{4.2}$$

Common matching technology then gives $f_B = f_G = f_U$ and $q_B = q_G$. Productivities, recruiting costs, and match values may differ to support different wages. The six aggregate targets, m, equal amenities, and equal tightness determine nine free parameters; the equilibrium branch and parameter count are documented in Appendix D. Equal tightness is an identifying restriction, not an observed feature of the two job types. No welfare outcome is imposed.

Economy C: A job ladder with on-the-job mobility

In C, type-B workers can accept more valuable type-G opportunities ($o > 0$). Beyond the common targets and m, the calibration matches $r = 0.003973$ and $\eta = 0.451768$, respectively the monthly rate of climbing per employed worker and the fraction of upper hires from nonemployment. Market tightness is now endogenous and need not be equal. We retain equal baseline amenities but allow productivities, recruitment costs, and preference parameters to adjust. The positive exposure convention $o = 0.01$ fixes units, not employed workers' time spent searching; its role is established in Section 4.4.

The comparison B→C is *target-matched*: both reproduce the aggregate targets and wage premium, while C also matches the two mobility moments. Their structural parameters need not coincide. We also compute *C with OTJ off*: starting from C's calibrated primitives, set only $o = 0$ and resolve the stationary equilibrium without restoring targets, equal tightness, or the output normalization. This is a fixed-primitives experiment, not economy B. Section 5 distinguishes their welfare implications.

4.3 Measuring the job ladder

We construct proxies for r, η, and m from archived public 2014–2019 Federal Reserve Bank of New York Survey of Consumer Expectations Labor Market Survey microdata. The model's permanent job types are not observed. We proxy the more valuable type-G job by overall satisfaction of four or five on the survey's five-point scale, and type B by responses one through three. This maintained mapping follows worker values rather than cash wages alone, but satisfaction categories are not literal permanent rungs.

Constructing the three moments

We link interviews exactly four months apart, identify new jobs using start dates between interviews (distinct from the prior start date), and classify employment and nonemployment using affirmative questionnaire routing rather than interpreting missing status as nonemployment. Each pair receives the average of its two positive cross-sectional survey weights, ω_j. Let C_s count weighted employed-origin moves from low to newly started high-satisfaction jobs, H_s count weighted nonemployment-to-high hires, and X_s count weighted respondents employed at the initial interview. The last denominator includes those who subsequently separate or have unclassified status; conditioning on continued employment would bias the exposure measure. For employed-origin upward moves with valid pay at both interviews, let $\mathcal{C}_{\text{pay}}$ be their set and $\text{earn}_{j,k}$ annual pretax earnings at interview k. The three proxies are:

$$\widehat{r} = \frac{C_s}{4X_s}, \tag{4.3}$$

$$\widehat{\eta} = \frac{H_s}{H_s + C_s}, \tag{4.4}$$

$$\widehat{m} = \frac{\displaystyle\sum_{j \in \mathcal{C}_{\text{pay}}} \omega_j \log\left(\frac{\text{earn}_{j,k}}{\text{earn}_{j,k-4}}\right)}{\displaystyle\sum_{j \in \mathcal{C}_{\text{pay}}} \omega_j}. \tag{4.5}$$

The first expression divides moves observed over four months by four to obtain a monthly proxy; it is not a directly observed monthly hazard. The second measures the origin mix of observed upper hires. The third is the weighted mean log earnings gain on an upward job change, including bonuses and other pretax pay. Details of earnings filters and alternative proxies are in Appendix F.

The unweighted sample contains 6,244 initially employed exposure pairs, 98 upward moves, 74 nonemployed-origin upper hires, and 88 upward moves with valid pay at both interviews. Weighted estimates are:

$$\widehat{r} = 0.003973, \qquad \widehat{\eta} = 0.451768, \qquad \widehat{m} = 0.111702. \tag{4.6}$$

Interpretation and limitations

We map these estimates to climbing per employed worker, the nonemployed-origin share of upper hires, and the two-rung log wage premium. The mapping is informative but imperfect: high satisfaction characterizes about 68.10 percent of survey employment versus 26.84 percent in calibrated C, and 18.19 percent of respondents with an unchanged job start date switch satisfaction categories. Four-month endpoints miss intervening moves; observed nonemployment may include nonsearchers; annual earnings may change with hours and payments; and the model omits high-to-high job switches, which are present in the survey. The three estimates are consequently *proxies under a maintained mapping*, not direct measurement of a permanent two-rung hierarchy. Appendix F documents construction, alternative definitions, and joint sampling uncertainty.

4.4 Recovering the ladder and normalizing exposure

What the employment flows identify

Let $N = n_B + n_G = (1-u)L$, $s = uL$, and $g = n_G/N$. Stationarity requires $\rho N = f_U s$. Since the type-B climbing probability is $\lambda = o f_G$, the climbing rate is $r = \lambda n_B/N$. The

nonemployed-origin share is $\eta = \alpha f_G s/(\rho n_G)$; its denominator includes both nonemployment hires and workers climbing from B. These flows imply:

$$\rho = \frac{f_U s}{N}, \tag{4.7}$$

$$g = \frac{r}{\rho(1-\eta)}, \tag{4.8}$$

$$\lambda = \frac{r}{1-g}, \tag{4.9}$$

$$c_\alpha \equiv \frac{\alpha}{o} = \frac{\eta}{1-\eta}\frac{n_B}{s}. \tag{4.10}$$

The aggregate moments identify separation and the upper employment share; climbing and hire origins identify the individual climbing probability and relative exposure c_α. The common upper-market contact rate cancels in the ratio of the two origin flows. Thus the moments identify α/o, not α and o separately.

Completing the calibration

For any admissible positive o, set $\alpha = c_\alpha o$ and recover:

$$f_G = \frac{\lambda}{o}, \qquad f_B = \frac{f_U - \alpha f_G}{1-\alpha}. \tag{4.11}$$

The aggregate vacancy target and common matching technology then determine matching efficiency, tightness, and vacancies in each market; upper tightness is not separately targeted. The wage premium, equal baseline amenities, and output normalization complete the compensation and resource calibration. Appendices C and F give inversion formulas and feasibility checks. After fixing the common parameters, C has eleven free primitives; eight empirical moments, $Y = 1$, and $a_B = a_G$ leave one exposure scale degree of freedom. We set $o = 0.01$, yielding $\alpha = 0.09605785$. This is an exposure-unit convention, not an estimate that employees spend one percent of their time searching.

Why the exposure normalization does not determine welfare

The identified objects are recruitment flows and their resource costs, not physical exposure and vacancy units separately. Write recruiting expenditures $E_i = \gamma_i v_i$. The matching functions can be expressed as:

$$M_B = \widetilde{\phi}_B E_B^{\xi} s^{1-\xi}, \tag{4.12}$$

$$M_G = \widetilde{\phi}_G E_G^{\xi} (n_B + c_\alpha s)^{1-\xi}, \tag{4.13}$$

where $\widetilde{\phi}_B = \phi \gamma_B^{-\xi} (1-\alpha)^{1-\xi}$ and $\widetilde{\phi}_G = \phi \gamma_G^{-\xi} o^{1-\xi}$. The upper-market search pool depends on identified relative exposure c_α, not on the absolute units of o. Changing a positive normalization *with consistent recalibration* leaves the expenditure-based technology, real allocation, and welfare comparisons unchanged within the common admissible probability domain (Appendix C). Switching mobility off also leaves a well-defined paired comparison: the upper pool becomes $c_\alpha s$.

By contrast, changing *only* o within C, without recalibrating other primitives, changes actual mobility opportunities and equilibrium outcomes. The zero-exposure intervention is not an alternative positive unit convention. Broad employed/nonemployed offer statistics do not isolate accepted upper-market contacts conditional on lower-rung employment and hence do not identify this scale (Appendix F).

5 From Identical Markets to A Mobile Job Ladder

5.1 Two exact aggregation results

Table 3 compares the reconstructed one-job economy of Epstein et al. (2023) (ENOP), economies A–C, and C with OTJ switched off. We report the ETI ϵ, its predicted normalized deadweight loss D_ϵ, actual normalized loss D, the ratio $R = D/D_\epsilon$, and loss in goods units H, as defined in Section 3.

Table 3: Stationary welfare along the benchmark sequence

Economy	ε	D_ε	D	R	H
One-type ENOP	0.790323	0.184181	0.197410	1.071822	0.166640
A: identical markets	0.790323	0.184181	0.197410	1.071822	0.166640
B: wage ladder, no OTJ	0.790323	0.184181	0.197410	1.071822	0.166640
C: OTJ ladder	0.790323	0.184181	0.199929	1.085504	0.165656
C with OTJ off	0.797527	0.185860	0.199280	1.072201	0.163397

$H = -W_\tau$ is the stationary flow loss in goods per unit increase in the tax fraction. $D = H/Z$ and $R = D/D_\varepsilon$. A/B/C each normalize $Y = 1$. The final row changes only o in C and retains the goods units of C; its output is .972687. The one-type entry uses the common reconstructed aggregate convention.

In ENOP, the ETI is 0.790323, $D_\epsilon = 0.184181$, and $D = 0.197410$, implying $R = 1.071822$: actual DWL exceeds the ETI prediction by about 7.18 percent. Economies A and B reproduce these aggregate results exactly, for different reasons.

In A, proportional replication divides employment, vacancies, search exposure, and amenity tastes across identical markets. Constant returns in matching preserves tightness and contact rates at each nearby tax rate; aggregate quantities and local tax derivatives are therefore unchanged. Splitting logarithmic amenity utility adds only a tax-independent constant. Appendix D proves the nesting for any admissible proportional split, not just the baseline $\alpha = 0.09605785$.

B introduces a genuine wage premium while maintaining common targets, $o = 0$, and equal tightness. Finding and filling rates remain equal; wages, productivity, recruiting costs, and installed-match values can differ by rung. To see why aggregate welfare accounting remains unchanged, let $N = (1-u)L$ and $\overline{J} = (1-\alpha)J_B + \alpha J_G$ be the employment-weighted mean firm value. Aggregating the firm and worker equations gives:

$$\overline{J} = \frac{Y}{N[(1+b)d_0 + b\beta f_U]}, \qquad K = \beta f_U s \overline{J}, \qquad \Pi = (1-\beta)N\overline{J}. \tag{5.1}$$

The mean installed-match value, recruiting expenditure K, and profits Π depend on aggregate objects, not the division of cash wages across rungs. Equal amenities per worker and

the common expenditure target then preserve the aggregate wage bill and welfare-relevant resources. This is not job relabeling: B's vacancy costs are 1.767829 and 5.025079, even though its tightness, finding, and filling rates coincide across markets.

Proposition 4 (Controlled wage-ladder invariance)**.** *Along the admissible equal-tightness calibration that introduces a cash-wage premium m while preserving the common aggregate targets, the aggregate stationary allocation and its local tax derivatives are independent of m. Consequently, the ETI, the ETI-based prediction D_ϵ, the stationary correction Ω, and DWL in goods units H are identical to those at $m = 0$.*

Proof. See Appendix D. □

This is a controlled invariance result, not a statement that arbitrary wage dispersion leaves welfare unchanged. It lets us identify what mobility adds beyond differentiated jobs.

The ETI prediction separates into participation and amenity-substitution contributions:

$$D_{\epsilon,L} = \frac{\tau}{t}\left(1 + \frac{A}{Z}\right)\frac{1-L}{\nu L}, \tag{5.2}$$

$$D_{\epsilon,A} = \frac{\tau}{t}\frac{A}{Z}. \tag{5.3}$$

Thus $D_\epsilon = D_{\epsilon,L} + D_{\epsilon,A}$ and $D = D_\epsilon + \Omega/Z$; Table 4 shows each component.

Table 4: Decomposition of normalized stationary welfare loss

Component	ENOP	A	B	C	C off
ETI: participation $D_{\epsilon,L}$	0.15855	0.15855	0.15855	0.15855	0.15996
ETI: amenity substitution $D_{\epsilon,A}$	0.02564	0.02564	0.02564	0.02564	0.02590
Total ETI prediction D_ϵ	0.18418	0.18418	0.18418	0.18418	0.18586
Additional correction Ω/Z	0.01323	0.01323	0.01323	0.01575	0.01342
Actual welfare loss D	0.19741	0.19741	0.19741	0.19993	0.19928

Components use each economy's own wage bill. $\Omega/Z = D - D_\epsilon$; displayed entries are rounded. ENOP/A/B/C share aggregate targets; C off changes only employed-worker exposure within C and does not restore them.

The common L, A/Z, and ν make the ETI contributions identical in ENOP/A/B/C, but

C's larger Ω/Z raises actual normalized DWL. In the structural OFF economy, participation and the amenity-to-wage ratio adjust, so both ETI contributions can change.

5.2 Adding mobility: two distinct comparisons

Table 5 distinguishes B→C (separate calibrations with common aggregate targets and wage premium) from C off→C (the same structural primitives except o). They should not be interpreted as the same intervention.

Table 5: Differences between stationary benchmarks

Comparison	$100\Delta R$	ΔH	$100\Delta H/H_{\text{initial}}$
ENOP → A	0.0000	0.000000000	0.0000
A → B	0.0000	0.000000000	0.0000
B → C	1.3681	-0.000984345	-0.5907
OTJ off → C	1.3302	0.002258892	1.3825

Every difference is destination minus origin. The first column of numbers is in percentage points of R; the second is in common goods units. The percentage in the last column uses the origin economy. B/C are target-matched calibrations, whereas the last row switches OTJ on (from the structural OFF economy to C), changing only OTJ exposure; the structural OFF economy is not B.

In B→C, actual DWL exceeds the ETI prediction by 7.18 percent in B and 8.55 percent in C, an increase of 1.3681 percentage points or 19.05 percent of B's correction. The ETI-based prediction is unchanged, yet the additional correction rises. Equation (3.26) makes its source clear:

$$D = \underbrace{\frac{\tau}{t}\epsilon}_{D_\epsilon} - (1+tb)\frac{\Pi}{Z}\frac{L_\tau}{L}.$$

Mobility changes employment composition, recruitment, and installed-match values, hence the profit-to-wage ratio Π/Z, even when the aggregate objects determining the ETI remain fixed. The result is not calibrated as a welfare target. Productivities, costs, and preferences adjust between B and C to accommodate mobility while preserving the common targets.

For the fixed-primitives experiment, set only $o = 0$ in calibrated C and resolve the equilibrium, keeping nonemployed exposure α and all other primitives unchanged. Both markets still hire from nonemployment, but workers cannot climb directly. C off is not B: its equilibrium amenity ratio becomes $a_G/a_B = 2.594915$, and the higher-value G job has a lower cash wage. Restoring mobility raises the correction from 7.22 to 8.55 percent, by 1.3302 percentage points, or 18.42 percent of the OFF correction. Unlike B→C, this comparison also changes participation and A/Z, and hence D_ϵ. Table 4 separates these channels.

For clarity, the welfare ratio has the normalization:

$$R - 1 = \frac{D - D_\epsilon}{D_\epsilon} = \frac{\Omega}{H_\epsilon}. \tag{5.4}$$

The reported 8.55 percent means that actual DWL exceeds the ETI prediction by that percentage *of the prediction*; the share of actual DWL omitted by the prediction instead equals $1 - 1/R$, or 7.88 percent for C. Neither measure alone determines DWL in goods units, to which we now turn.

5.3 The correction and welfare loss in goods

Since $H = H_\epsilon + \Omega$ and $H_\epsilon = ZD_\epsilon$, a larger R can coexist with a smaller goods-unit loss: wage bills can differ even when the normalized ETI prediction is the same. In B→C, the goods-unit change is:

$$\Delta H = \underbrace{-0.002866324}_{\Delta H_\epsilon} + \underbrace{0.001881979}_{\Delta\Omega} = -0.000984345. \tag{5.5}$$

DWL in goods falls by 0.591 percent because the decrease in the ETI-predicted loss dominates the larger correction. Although D_ϵ is unchanged, the wage bill relative to output falls from $Z/Y = 0.844135$ in B to 0.828572 in C (both have $Y = 1$), reducing $H_\epsilon = ZD_\epsilon$. Figure 1 contrasts losses relative to each economy's wage bill with their values in common goods units.

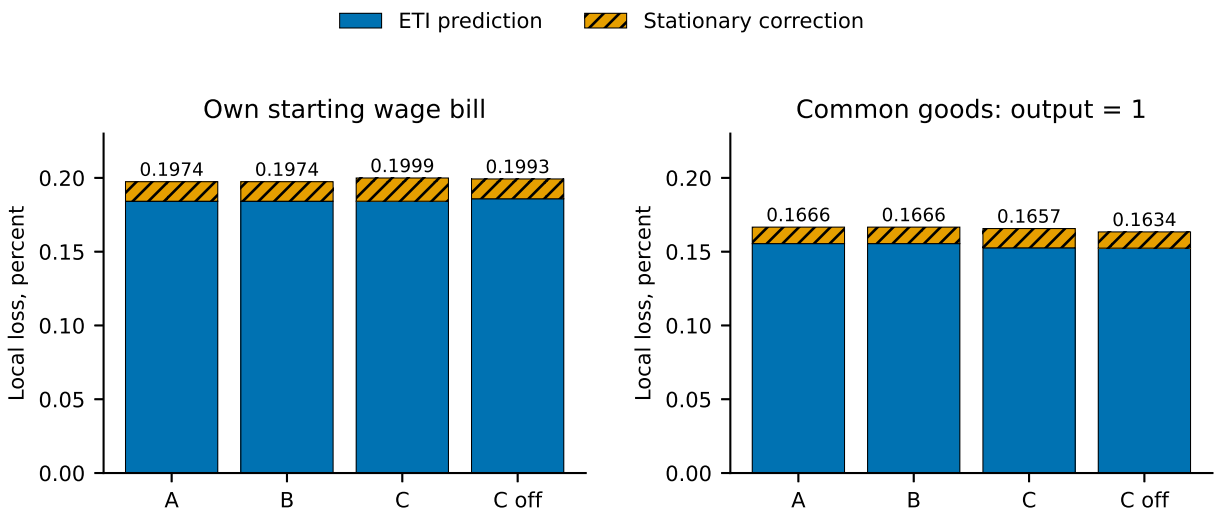


Figure 1: Local stationary tax losses across the benchmarks

Bars show $.01H$ as a percentage of each economy's initial wage bill (left) or common baseline output $Y = 1$ (right). The solid component is the ETI prediction; the hatched component is the stationary correction. A/B/C are target-matched. C and C off differ only in o; OFF output is not renormalized. These are local stationary-flow welfare comparisons.

Restoring mobility from C off instead increases both goods-unit components:

$$\Delta H = \underbrace{0.000213465}_{\Delta H_\epsilon} + \underbrace{0.002045427}_{\Delta\Omega} = 0.002258892. \tag{5.6}$$

Goods-unit DWL rises from 0.163397 to 0.165656, or 1.382 percent. Here both Z and D_ϵ change; most of the increase comes from $\Omega = -(1+tb)(\Pi/L)L_\tau$. Restoring mobility raises the upper employment share from 0.123869 to 0.268403 and profits per participant Π/L by 19.16 percent. An exact midpoint decomposition of $\Omega = A_c X$, with $A_c = -(1+tb)L_\tau$ and $X = \Pi/L$, attributes 0.002103551 of the change to the profit-density component and -0.000058124 to the participation-response component. This is accounting for a joint equilibrium change, not two separate interventions.

C off retains the goods units of calibrated C: its output is 0.972687, not renormalized to one. The two experiments consequently increase the additional welfare correction but have

opposite effects on total goods-unit loss. Appendix G reports the numerical welfare checks.

5.4 Productive opportunities and the welfare comparison

To vary actual mobility opportunities rather than calibration targets, we change only o within C, holding all other structural primitives fixed. Over the reported eight-point grid from zero to 0.02 (baseline 0.01), climbing, participation, compensation, and ETI respond endogenously; $\lambda = of_G$ need not change proportionally with o. Figure 2 displays the resulting welfare correction and goods-unit DWL.

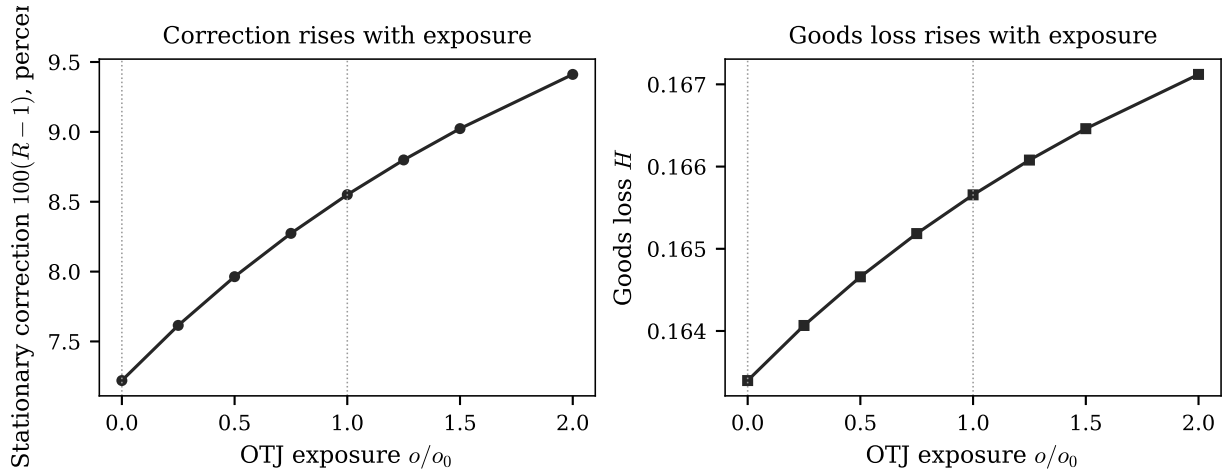


Figure 2: OTJ exposure and stationary welfare, all other primitives fixed
Each point varies only o from C ($o_0 = .01$); participation, climbing, and ETI adjust endogenously. Goods units remain those of C. Left: $100(R-1)$; right: H. Dotted lines mark OFF and baseline. Values are in Appendix Table A12.

Both measures increase along this grid: (R, H) moves from $(1.072201, 0.163397)$ at $o = 0$, to $(1.085504, 0.165656)$ at baseline, and $(1.094112, 0.167121)$ at $o = 0.02$. This is a result for the reported fixed-primitives grid, not a global monotonicity theorem. Crucially, changing only o is not a positive *renormalization* of exposure: the latter requires compensating changes in α, matching, and recruitment-cost parameters that leave the expenditure-space technology and real allocation invariant (Section 4.4). Setting $o = 0$ instead removes climbing altogether.

The exposure experiment is also distinct from the planner comparison in Section 3.1. Recruiting an employed B worker to G destroys an incumbent B match, a displacement the

private recruiter need not internalize. Appendix B constructs a feasible deviation that reduces upper-market vacancies for one month by 10 percent and increases discounted welfare, holding aggregate participation and amenity expenditure fixed. That finding identifies recruitment inefficiency; it does *not* measure the welfare effect of abolishing mobility, which would also eliminate opportunities to reach more valuable jobs. The benchmark, exposure, and planner exercises answer different questions.

6 Compensation, Identification, and Sensitivity

6.1 What changes across the compensation experiments

The baseline imposes equal amenities per worker, an assumption not observed in the job ladder. We consider two distinct departures. First, we redistribute a fixed total amenity taste across job types without changing other primitives. Second, we vary relative amenities per worker and recalibrate latent primitives to preserve the maintained observations. The first exercise isolates compensation composition; the second asks whether those observations identify the welfare correction. Appendix Table A11 distinguishes these exercises from changes in actual mobility opportunities and from changes in exposure units.

For the first exercise, fix total amenity taste $S_a = \zeta_B^a + \zeta_G^a$ and assign share $\sigma = \zeta_G^a / S_a$ to the upper rung, so that $\zeta_B^a = (1-\sigma)S_a$ and $\zeta_G^a = \sigma S_a$, with $0 < \sigma < 1$. Holding the other primitives fixed leaves the reduced matching and match-value system, and hence participation and employment, unchanged on a common admissible equilibrium branch. Aggregate amenity expenditure depends only on total taste, $A = S_a/t$. With effective compensation $e_i = w_i + a_i$, the compensation conditions imply

$$a_i = \frac{\zeta_i^a}{t n_i}, \qquad w_i = e_i - a_i, \qquad Z = \sum_i e_i n_i - \frac{S_a}{t}. \tag{6.1}$$

Thus cash wages and amenities are redistributed across rungs, but the taxable wage bill,

aggregate resources, and their nearby-tax responses do not change. The ETI prediction, additional stationary correction, welfare ratio R, and goods-unit loss H are invariant. Utility levels can differ across taste specifications by a tax-independent constant. Proposition 5 in Appendix E proves the result with and without OTJ and states the positive-pay and ranking restrictions. In particular, cash-wage rankings can reverse without reversing the worker-value ranking that supports climbing.

The second exercise need not preserve the resource allocation. It varies amenities per worker while recalibrating productivities, recruiting costs, and preferences to the same aggregate and mobility moments. Its no-OTJ counterpart switches off exposure within each calibrated economy; it is an equilibrium prediction, not a separately target-matched calibration.

6.2 Admissible target-preserving latent calibrations

Let $k = a_G/a_B$ denote amenities per upper-rung worker relative to a lower-rung worker. For each selected k, we preserve participation, unemployment, nonemployed job finding, vacancies per nonemployed searcher, normalized output, the amenity-to-wage expenditure ratio, the cash-wage premium $m = \log(w_G/w_B)$, the climbing rate r, and the nonemployed-origin share of upper hires η. The baseline sets $k = 1$; these observations do not independently measure k.

The target $\omega = A/Z$ determines amenities conditional on k. Writing $x = \exp(m) = w_G/w_B$ and letting $g = n_G/(n_B+n_G)$ denote upper-rung employment's share, temporary compensation units $w_B^* = 1$, $w_G^* = x$ give

$$a_B^* = \omega \frac{1 - g + xg}{1 - g + kg}, \qquad a_G^* = k a_B^*. \tag{6.2}$$

For each allocation we solve the inverse calibration for the latent primitives and normalize output to $Y = 1$; Appendix C gives the formulas. Unlike fixed-total-taste redistribution, recalibration can change taxable wages Z, amenities A, recruiting expenditure K, and profits

Π in final-good units. Fixing the relative cash wage does not fix cash-wage levels: the output normalization can change the common compensation scale. The maintained observations restrict ratios and flows, not the full allocation of output. The economies nevertheless have the same model-implied ETI, 0.7903, because L, A/Z, and participation curvature ν are common by (3.17). Equal ETI here follows from the targets; it is not an independent additional restriction.

Figure 3 and Appendix Table A19 show that these observationally equivalent calibrations imply different stationary corrections: 7.32 percent at $k = 0.25$ and 10.73 percent at $k = 4$. Within each calibration we construct a structural OFF counterpart by setting only $o = 0$, resolving equilibrium, and retaining the ON economy's goods units. OFF participation, compensation, ETI, and resource allocation may adjust. At $k = 4$, the ON correction of 10.73 percent exceeds its paired OFF correction of 7.63 percent by 3.1025 percentage points, or 40.65 percent of the OFF correction.

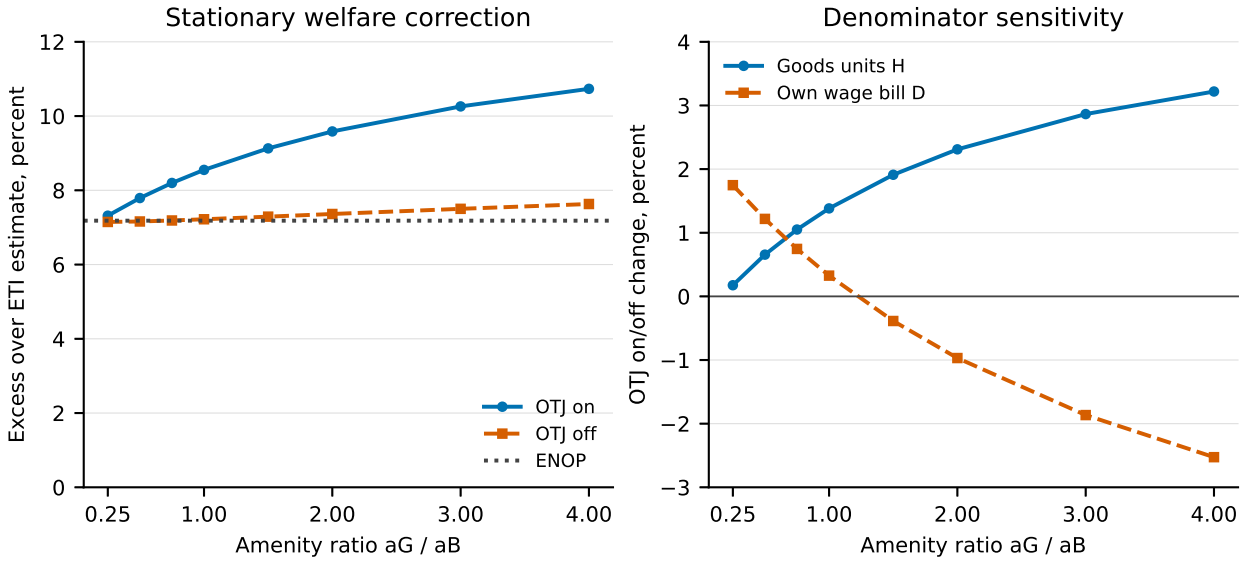


Figure 3: Admissible target-preserving latent calibrations

The ON economies retain the common targets, cash-wage premium and implied ETI while $k = a_G/a_B$ varies from .25 to 4. Each paired OFF economy changes only o and retains its ON parent's goods units. The right panel reports relative OFF-to-ON changes in H and $D = H/Z$, using each OFF value as denominator. The .10 point is inadmissible. The lines use distinct styles and markers as well as color.

The welfare identity pinpoints the missing information:

$$D - D_\epsilon = -(1 + tb)\frac{\Pi}{Z}\frac{L_\tau}{L}.$$

The factor multiplying Π/Z is common across target-preserving ON calibrations. Their different profit-to-wage ratios therefore produce different corrections even though their observed labor-market moments and ETIs coincide. This is a lack of identification conditional on the declared observations and calibrated profile: changing the unmeasured amenity allocation changes the implied profit share without changing the fitted moments. The result concerns the correction relative to the ETI prediction, not a proportional change in welfare loss in goods. Across the reported ON profile, H runs from 0.165341 to 0.166199, a 0.519-percent increase.

The profile includes only admissible equilibria: cash wages and match values are positive, matching probabilities feasible, and the worker-value ranking supports climbing. The strict-ranking boundary is $k_* = 0.186141$; the proposed $k = .10$ point fails, whereas all reported points in $[.25, 4]$ pass the checks. Thus the correction at $k = 4$ is the endpoint of our declared profile, not a global upper bound or a confidence limit. The paired normalized-loss difference $D_{\text{on}} - D_{\text{off}}$ crosses zero at $k = 1.2155$, although the goods-unit increment remains positive. As in Section 5.3, different equilibrium wage bills can change the sign of normalized differences without reversing the goods comparison or eliminating either economy's correction.

6.3 The resource information that distinguishes the calibrations

What additional observation would distinguish these latent economies? The resource identity is

$$1 = \frac{Z}{Y} + \frac{A}{Y} + \frac{K}{Y} + \frac{\Pi}{Y}. \tag{6.3}$$

The calibration fixes $A/Z = \omega$, implying $1 = (1 + \omega)Z/Y + (K + \Pi)/Y$, but does not fix the individual resource shares. A compatible observation of the taxable wage bill relative

to model output, Z/Y, supplies resource information absent from the maintained moments. Along our one-dimensional profile it selects k, and with it the associated welfare correction.

To see why, use temporary wage units to define $\mathcal{Z} = n_B + xn_G$ and the upper-rung share of amenity spending $\chi(k) = kn_G/(n_B + kn_G)$. The employment levels, relative cash wage, $\mathcal{Z}$, and total temporary amenity expenditure $\omega\mathcal{Z}$ remain fixed along the profile. The firm-value system makes temporary output affine in χ, yielding

$$\frac{Z}{Y}(k) = \frac{\mathcal{Z}}{\mathcal{Y}_0 + \mathcal{Y}_1\chi(k)}, \qquad \mathcal{Y}_0 > 0, \quad \mathcal{Y}_1 > 0. \tag{6.4}$$

As amenities move toward the upper rung, the recalibrated firm values and productivities support higher temporary output even though temporary cash compensation is unchanged. Normalization to $Y = 1$ does not change their ratio. Since $\chi'(k) = n_B n_G/(n_B + kn_G)^2 > 0$ and the calibrated $\mathcal{Y}_1$ is positive, Z/Y decreases strictly with k. This sign holds for the maintained calibration, not necessarily for all parameterizations; Appendix E derives the coefficients.

Consequently, a compatible exact observation z^{obs} can be inverted:

$$\chi^{\text{obs}} = \frac{\mathcal{Z}/z^{\text{obs}} - \mathcal{Y}_0}{\mathcal{Y}_1}, \qquad k^{\text{obs}} = \frac{n_B\chi^{\text{obs}}}{n_G(1 - \chi^{\text{obs}})}. \tag{6.5}$$

The selected k determines a calibrated ON economy and its paired structural OFF prediction. An observation outside the attainable range rejects this profile; if k were already known independently, Z/Y would instead be an overidentifying restriction. Neither the resource identity by itself nor this conditional inverse identifies unrestricted compensation heterogeneity.

Figure 4 illustrates the mapping and an example of measurement precision. Across $k \in [.25, 4]$, the attainable ON ratio is $[0.814899, 0.836503]$. A hypothetical half-width of 0.005 around the baseline ON value 0.828572 selects $k \in [0.4876, 1.7170]$, and hence a paired mobility increment $100(R_{\text{on}} - R_{\text{off}})$ between 0.6105 and 2.0185 percentage points. This is a conditional precision illustration, not an estimated empirical confidence interval; Appendix E

reports the additional checks and bands. Welfare bounds over a selected interval use extrema along that interval rather than assuming that every welfare object is monotone; Appendix E documents the relevant interior extremum.

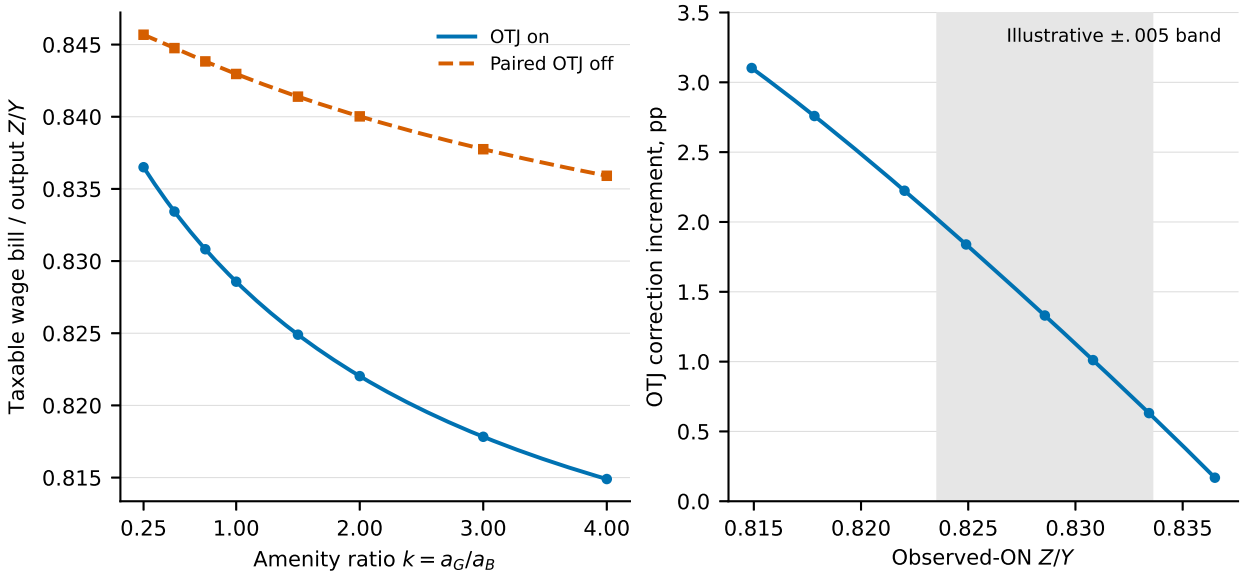


Figure 4: Resource accounting and the latent calibration profile

The left panel reports each economy's own Z/Y. The right panel uses the ON economy's observable to select a calibrated pair and plots $100(R_{\text{on}} - R_{\text{off}})$. The shaded interval is a hypothetical measurement-precision band of $\pm.005$ around baseline ON Z/Y. It is not an estimated empirical band. Both panels retain $k \in [.25, 4]$ and use the validated continuous calibration profile.

The proposed additional observation must match the model's resource boundary. Here Z is taxable cash compensation and Y is output available for cash wages, goods-cost amenities, recruitment, and match profits. The model omits physical capital, depreciation, and production taxes. A conventional labor share with a different compensation basket or output boundary is therefore not automatically the required Z/Y. Capital-service charges, noncash pay, recruiting expenditure, and sector and period coverage would need reconciliation before using an empirical moment. We identify the information required, not claim that it has already been independently measured.

6.4 Lower ETI and structural parameters

The preceding results are conditional on the calibration. We examine a lower model-implied ETI and alternative matching elasticity, discounting, and amenity expenditure; these are parameter scenarios, not new parameter estimates. To lower the ETI while preserving the initial OTJ-on allocation, we increase participation curvature ν and adjust the home-utility scale to keep marginal home utility unchanged. Writing $\omega = A/Z$, the ETI decomposition in (3.17) implies

$$\nu = \frac{(1+\omega)(1-L)}{L(\varepsilon-\omega)}, \qquad \varepsilon > \omega. \tag{6.6}$$

This adjustment changes the nearby-tax response, not the initial employment or compensation allocation. Each structural OFF economy is then resolved with its own calibrated primitives and $o = 0$, without independent target matching.

Table 6: Sensitivity of the OTJ-on stationary ratio to parameter scenarios

Scenario	Value	R_{on}
Baseline	model-implied ETI 0.7903	1.085504
Lower ETI	$\varepsilon = .40$	1.072013
	$\varepsilon = .30$	1.062908
	$\varepsilon = .20$	1.044698
Matching elasticity	$\xi = .4$	1.069766
	$\xi = .6$	1.109110
Discounting	$\beta = .9999$	1.002189
Amenity expenditure	$A/Z = .05$	1.093010
	$A/Z = .20$	1.077156

Each scenario retains the maintained mobility calibration and reports the OTJ-on stationary ratio $R_{\text{on}} = D/D_{\varepsilon}$. Lower-ETI rows change participation curvature ν with the home-utility scale adjusted to preserve baseline marginal utility; the ε and A/Z rows also change the ETI. Every admitted case retains a positive stationary correction ($R_{\text{on}} > 1$). These are parameter scenarios, not statistical uncertainty; full grids are in Appendix G.

The OTJ-on welfare ratio falls from 1.072013 at $\varepsilon = .40$ to 1.044698 at $\varepsilon = .20$, but the correction remains positive: at the latter elasticity, actual DWL exceeds the ETI prediction

by 4.47 percent. Within that same scenario, restoring OTJ raises the correction by 0.71 percentage points and goods-unit loss by 0.9446 percent. Curvature alone cannot lower the ETI below $\omega = .11$, its amenity-substitution component.

Matching elasticity changes contact and vacancy conditions, whereas discounting changes the link between installed-match values and the profit flow in (2.15). At $\beta = .9999$, for example, $R = 1.002189$; that result includes equilibrium adjustments in match values. Changing A/Z also changes the ETI and the allocation of resources, unlike the fixed-total-taste experiment above. Every admitted scenario in Table 6 has $R > 1$, but their magnitudes do not establish a universal sign or a monotonic response across all parameterizations. Appendix G reports the full grids.

6.5 Alternative rung measurements and joint bootstrap

The baseline maps job-satisfaction responses into model rungs to estimate (r, η, m), but satisfaction is not a direct measure of permanent job type. We therefore consider four classifications: overall satisfaction ≥ 4 (baseline), overall satisfaction $= 5$, nonwage satisfaction ≥ 4, and nonwage satisfaction $= 5$. Each produces a new calibrated ON economy under the common aggregate targets and equal baseline amenities; its OFF counterpart removes only o. Table 7 reports the point comparisons.

Table 7: Welfare comparisons across rung proxies

Proxy	R_{on}	ΔR, pp	ΔH, %	ΔD, %
Overall ≥ 4	1.085504	1.3302	1.3825	0.3260
Overall $= 5$	1.088937	1.6632	1.7290	-0.2508
Nonwage ≥ 4	1.085430	1.3174	1.3691	-0.0047
Nonwage $= 5$	1.078725	0.6864	0.7131	0.4042

Benchmark aggregate targets and equal baseline amenities. ΔR means $100(R_{\text{on}} - R_{\text{off}})$; ΔH and ΔD are relative percentage changes.

Under every proxy, restoring mobility raises both the welfare ratio and DWL in goods units. The ratio increment ranges from 0.6864 to 1.6632 percentage points and the goods-unit

increase from 0.7131 to 1.7290 percent. Changes in $D = H/Z$ take both signs because the paired wage bills differ. These checks address sensitivity to the job-type proxy, not whether satisfaction categories actually identify permanent job rungs.

For sampling variation, joint respondent resampling reconstructs (r, η, m) together rather than treating their estimates as independent. There are 999 draws for each proxy under each of two aggregate conventions (7,992 attempted calibrations). We retain failures in the replication output and admit only pairs with positive cash pay and match values, strict worker ranking, and feasible probabilities. Admission counts are 992, 999, 994, and 905 out of 999 by proxy, respectively; the final proxy has 81 ranking failures and 13 additional OFF cash-pay failures. Appendix Table A16 gives the proxy-specific outcomes.

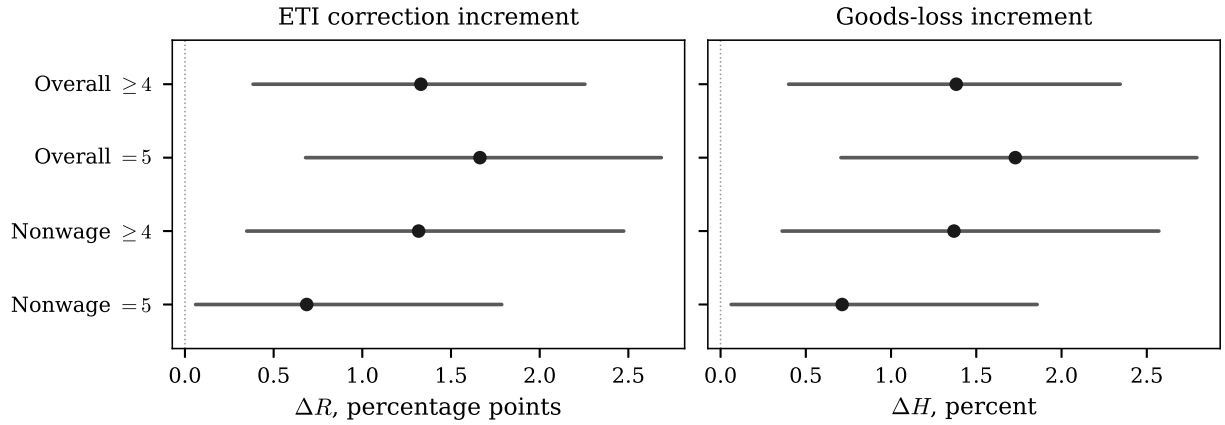


Figure 5: Joint proxy uncertainty conditional on admission

Points are original-sample estimates; segments are 2.5th–97.5th percentiles among admitted joint respondent draws. *Left:* the ETI correction increment ΔR in percentage points. *Right:* the goods-loss increment ΔH in percent. Both are positive throughout the admitted draws. The normalized-D ranges, whose sign varies across draws, are retained in Appendix Table A16. Benchmark aggregate targets. These ranges are conditional proxy-bootstrap percentiles, not unconditional or official survey-design confidence intervals.

Every admitted draw has $R_{\text{on}} > R_{\text{off}}$ and $H_{\text{on}} > H_{\text{off}}$. For the baseline proxy, the conditional 2.5th–97.5th percentile interval for the increase in R is $[0.38, 2.26]$ percentage points; for the relative increase in goods loss it is $[0.3986, 2.3441]$ percent. The corresponding change in normalized D lies in $[-1.1100, 1.6928]$ percent, so a positive goods-unit increment need not imply a positive normalized one.

These are conditional percentiles among model-admitted draws, not unconditional or

official survey-design confidence intervals. They do not establish a universal sign across structural parameterizations, nor can resampling settle whether the satisfaction proxy represents stable rungs. The stock/flow and within-spell reclassification limitations in Section 4.3 remain. The identification result above concerns missing resource information conditional on the model; proxy and bootstrap exercises concern the measurement of its mobility inputs.

7 Conclusion

Does a job ladder change what the elasticity of taxable income (ETI) reveals about the welfare cost of taxation? Our sequential comparisons show that duplicating a recruitment market leaves the existing ETI–DWL gap unchanged. A controlled wage hierarchy without direct job-to-job moves also leaves it unchanged. Mobility through on-the-job search is the relevant departure: recruiting an employed worker displaces an incumbent match and changes resource use and installed-match values in ways that taxable-income responses do not fully capture. In the target-matched comparison, adding mobility raises the preexisting welfare correction by approximately 19 percent.

That relative increase is not the same as an increase in welfare loss measured in goods. Our target-matched B-to-C comparison raises the relative correction while slightly reducing goods-unit loss; restoring mobility within C's fixed primitives raises both. These are different experiments, not conflicting results.

The magnitude is also conditional on compensation that standard labor-market moments do not identify. Alternative target-preserving calibrations with different amenities across rungs reproduce the same maintained observations and ETI yet imply different corrections. At the upper endpoint of our admissible profile, the paired OTJ increment is approximately 41 percent. A compatible taxable-wage-bill-to-output observation can distinguish calibrations within that profile. Our survey classifications discipline the illustration but do not directly identify permanent job types. Thus mobility matters for what taxable income omits, and the

allocation of resources across wages, amenities, and recruiting matters for measuring how much it omits.

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A Household optimization and equilibrium

A.1 Dynamic household and firm problems

At date k, employment (n_B, n_G) is inherited. The household's Bellman equation is

$$V_k(n_B, n_G) = \max_s \Big\{ t_k(w_{B,k} n_B + w_{G,k} n_G) + \Pi_k + T_k + h(1 - s - n_B - n_G) + \sum_i \zeta_i^a \log(a_{i,k} n_i) + \beta V_{k+1}(n'_B, n'_G) \Big\}, \tag{A.1}$$

subject to the private employment laws (2.1)–(2.2). The household takes contact rates, dividends, transfers and contracts as given. Its own search or marginal employment does not change an aggregate rate. Although symmetry ultimately requires $T_k = \tau_k Z_k$, the private search derivative does not differentiate that aggregate transfer rule.

Let $\delta_{i,k} = \partial V_k / \partial n_i$. The search condition is

$$h'(\ell_k) = \beta[(1 - \alpha) f_{B,k} \delta_{B,k+1} + \alpha f_{G,k} \delta_{G,k+1}]. \tag{A.2}$$

The envelopes, holding individual per-worker contracts fixed, are

$$\delta_{B,k} = t_k w_{B,k} + \frac{\zeta_B^a}{n_{B,k}} - h'(\ell_k) + \beta[(1 - \rho - \lambda_k)\delta_{B,k+1} + \lambda_k \delta_{G,k+1}], \tag{A.3}$$

$$\delta_{G,k} = t_k w_{G,k} + \frac{\zeta_G^a}{n_{G,k}} - h'(\ell_k) + \beta(1 - \rho)\delta_{G,k+1}. \tag{A.4}$$

In particular, the lower employment envelope includes the probability of reaching the upper type. No separate incumbent-value subtraction is added inside the destination bargain: the quit-before-bargaining convention already specifies its nonemployment disagreement state.

With $R_{B,k} = 1 - \rho - \lambda_k$ and $R_{G,k} = 1 - \rho$, a type-i firm solves

$$\mathcal{F}_{i,k}(n) = \max_{v \geq 0} \left\{ (y_i - w_{i,k} - a_{i,k})n - \gamma_i v + \beta \mathcal{F}_{i,k+1}(R_{i,k} n + q_{i,k} v) \right\}. \tag{A.5}$$

Linearity implies an installed-match value independent of the firm's size:

$$J_{i,k} = y_i - w_{i,k} - a_{i,k} + \beta R_{i,k} J_{i,k+1}, \qquad \gamma_i = \beta q_{i,k} J_{i,k+1}. \tag{A.6}$$

These conditions use the separation timing, entry lag, and household preferences specified in Section 2.

A.2 Financial accounting in the general dynamic environment

Before the quasilinear specialization, the household chooses consumption, search, and one-period bond holdings subject to its budget constraint in Section 2.1 and the employment transitions. A type-i firm can issue debt $\mathcal{D}_{i,t}$ at gross return R^b_t. Its dividend flow is

$$\Pi_{i,t} = (y_i - w_{i,t} - a_{i,t}) n_{i,t} - \gamma_i v_{i,t} + \mathcal{D}_{i,t} - R^b_{t-1} \mathcal{D}_{i,t-1}.$$

Let $\Xi_{t|0}$ be the household discount factor from date t to the initial date. The firm chooses vacancies and debt to maximize

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \Pi_{i,t},$$

subject to its employment transition and the usual no-Ponzi condition. Asset-market clearing requires

$$\mathcal{B}_t = \mathcal{D}_{B,t} + \mathcal{D}_{G,t}.$$

These financial flows cancel in the aggregate resource constraint. Quasilinearity equates the household's and firm's discounting at β; asset positions are then irrelevant for the benchmark allocations and can be normalized to zero. The Bellman equations above present that

benchmark specialization.

A.3 Individual deviations before symmetry

Consider a current individual wage deviation at fixed aggregate rates, future contracts and acceptance regime. The derivative of the worker payoff is t_k and that of the firm payoff is -1. Strict next-period upper-value ranking and contract history independence keep the move decision unchanged. Exogenous o also eliminates an individual effort response. Thus $\partial\lambda_k/\partial w_{B,k} = 0$ under this deviation.

For an atomistic current amenity bundle a_{match}, the relevant derivative is

$$\frac{\partial\delta_{i,k}}{\partial a_{\text{match}}} = \frac{\zeta_i^a}{A_{i,k}}, \qquad \frac{\partial J_{i,k}}{\partial a_{\text{match}}} = -1. \tag{A.7}$$

The marginal utility of household amenity services is evaluated before symmetry. Differentiating the already symmetric employment-envelope term ζ_i^a/n_i with respect to the individual amenity would incorrectly remove this derivative.

The two first-order conditions from the log Nash objective are

$$\frac{\psi t_k}{\delta_{i,k}} = \frac{1-\psi}{J_{i,k}}, \qquad \frac{\psi\zeta_i^a}{\delta_{i,k}A_{i,k}} = \frac{1-\psi}{J_{i,k}}. \tag{A.8}$$

Their combination gives $\delta_{i,k} = t_k b J_{i,k}$ and $A_{i,k} = \zeta_i^a/t_k$, where $b = \psi/(1-\psi)$. On the positive-surplus set the objective is concave in the affine individual payoffs. At the equilibrium marginal utility, cash wages and amenities have the same pretax value to the worker and the same unit resource cost to the firm. The bilateral objective therefore has a flat composition direction. Symmetric consistency $A_i = a_i n_i = \zeta_i^a/t$ fixes total services; the firm value fixes effective pay and hence symmetric cash pay. The argument establishes the maintained equilibrium conditions without claiming a unique individual package under strict bilateral concavity.

A.4 Stationary system and the one-type limit

The thirteen stationary equations are: the two stock equations (2.7)–(2.8), participation (2.9), the two household envelopes (2.10)–(2.11), the two firm envelopes (2.12), two free-entry conditions (2.13), and two wage-sharing and two amenity conditions in (2.14). Home time, matching rates, amenities, resource use, profits and transfers are definitions rather than additional restrictions. Equal baseline amenity levels are not an extra equilibrium equation.

The one-type limit has unknowns $(s, n, v, w, a, \delta, J)$ and equations

$$\begin{aligned} &\rho n = fs, && h' = \beta f \delta, && d_0 \delta = tw + \zeta_a/n - h', \\ &d_0 J = y - w - a, && \gamma = \beta q J, && \delta = tbJ, && an = \zeta_a/t. \end{aligned} \tag{A.9}$$

It removes the upper amenity utility term and sets the surviving search pool to s. In contrast, the two-type off economy retains $\alpha > 0$ and positive upper amenities. Its seven common classes of conditions remain the original thirteen scalar equations with $o = 0$.

A.5 Proof of tax invariance and the ETI expression

Substitute $a_i n_i = \zeta_i^a / t$ and $\delta_i = tbJ_i$ into the household envelopes and combine them with firm values. This gives (3.9)–(3.12), with no direct tax term after defining $Q = h'/t$. Free entry expresses J_i as a function of θ_i. The nonsingular two-by-two reduced Jacobian therefore implies locally constant tightness under a tax change. Matching probabilities, J_i and Q are also constant. The firm envelopes imply constant effective pay e_i.

The stationary stock laws then fix n_i/s and v_i/s, so all masses scale with L. The household participation condition $h'(1 - L) = tQ$ gives $L_\tau = Q/h''(1 - L) = -(1 - L)/(\nu t)$. Moreover,

$$A_\tau = \frac{A}{t}, \qquad Z_\tau = \frac{Z + A}{L} L_\tau - \frac{A}{t}. \tag{A.10}$$

Multiplying by $-t/Z$ proves (3.17). The generalized home utility can be written as $\zeta(\ell^{1-\nu} -$

$1)/(1-\nu)$ for $\nu \neq 1$ and $\zeta \log \ell$ for $\nu = 1$; the additive normalization has no effect on the derivatives.

A.6 Proof of the welfare identity

Stationary lower replacement hiring is $(\rho+\lambda)n_B$ and upper hiring is ρn_G. Free entry gives the recruiting bills $\beta J_B(\rho+\lambda)n_B$ and $\beta J_G \rho n_G$. Subtracting them from firm operating surplus leaves $(1-\beta)\sum_i J_i n_i$, proving (2.15).

Next multiply each household envelope by its employment mass and add. Use both stock equations to replace net continuation flows, and use participation to value the nonemployed search flows. The result is

$$t(Z+A) - h'L = (1-\beta)\sum_i \delta_i n_i = tb\Pi. \tag{A.11}$$

This intermediate identity includes the search mass in L. Omitting it would give an incorrect welfare correction.

Since Y and K scale with L, and since $\zeta_i^a/A_i = t$, differentiation of stationary resource welfare gives

$$W_\tau = \left[\frac{Y-K}{L} - h'\right] L_\tau - \frac{\tau A}{t}. \tag{A.12}$$

Subtract τZ_τ using (A.10). With $Y - K = Z + A + \Pi$ and (A.11),

$$\begin{aligned} W_\tau - \tau Z_\tau &= \left[\frac{t(Z+A)+\Pi}{L} - h'\right] L_\tau \\ &= [1+tb]\frac{\Pi}{L} L_\tau. \end{aligned} \tag{A.13}$$

Changing signs and using $H = -W_\tau$ and $H_\varepsilon = -\tau Z_\tau$ proves (3.25); substitution of the participation derivative and $H_\varepsilon = \tau Z \varepsilon/t$ proves (3.28). Positivity follows under the proposition's stated interior conditions. As β approaches one with bounded installed values the correction can approach zero. As ν grows while baseline levels and positive A/Z are preserved, it also

approaches zero. Neither limiting case creates an interior sign change.

B Planner derivation and implementation

B.1 Feasible choices and origin composition

Let $r(x)$ be current resource welfare at $x = (n_B, n_G, s, v_B, v_G, A_B, A_G)$:

$$r(x) = \sum_i y_i n_i - \sum_i A_i - \sum_i \gamma_i v_i + h(1 - s - n_B - n_G) + \sum_i \zeta_i^a \log A_i. \tag{B.1}$$

The planner maximizes $r(x) + \beta P(n'_B, n'_G)$ with fixed α, o, population feasibility and the same employment transitions. In matching notation those transitions are

$$n'_B = (1-\rho)n_B + M_B - pM_G, \qquad n'_G = (1-\rho)n_G + M_G, \qquad p = \frac{on_B}{S_G}. \tag{B.2}$$

The planner does not condition an upper contact on its preferred origin. If acceptance is a planner control, replace the employed-origin upper flow by $x_a p M_G$ and its subtraction from B by the same term, with $x_a \in [0,1]$. Full acceptance is locally optimal when $\mu_G > \mu_B$, and nonemployed hires are beneficial when $\mu_i > 0$. These inequalities are verified at the reported planner candidates. They are not consequences of private worker ranking alone.

At a stationary candidate write

$$\mathcal{V} = \mu_G - p\mu_B, \qquad \mathcal{A} = (1-\xi)\mathcal{V} - (1-p)\mu_B, \qquad \mathcal{B} = \mu_G - \xi\mathcal{V}. \tag{B.3}$$

The upper continuation contribution is $M_G\mathcal{V}$. Its derivatives with respect to search and lower employment are, respectively,

$$\alpha f_G[(1-\xi)\mathcal{V} + p\mu_B] = \alpha f_G \mathcal{B}, \qquad o f_G[(1-\xi)\mathcal{V} - (1-p)\mu_B] = o f_G \mathcal{A}. \tag{B.4}$$

Both include the change in the origin composition of upper hires.

Let κ denote the marginal opportunity cost of participation. For the unrestricted resource planner $\kappa = h'$. The stationary conditions are

$$\begin{aligned} \gamma_B &= \beta\xi q_B \mu_B, & \gamma_G &= \beta\xi q_G \mathcal{V}, \\ \kappa &= \beta[(1-\xi)(1-\alpha) f_B \mu_B + \alpha f_G \mathcal{B}], \\ d_0 \mu_B &= y_B - \kappa + \beta o f_G \mathcal{A}, & d_0 \mu_G &= y_G - \kappa. \end{aligned} \tag{B.5}$$

Unrestricted amenity choice gives $A_i = \zeta_i^a$. These conditions, together with stationarity and the resource constraint, completely specify the interior stationary planner system. The planner has no first-order condition for α or o because they are primitives.

B.2 Zero-tax no-OTJ optimality from common initial stocks

When $\tau = o = 0$ and $\psi = 1 - \xi$, define $\mu_i = \delta_i + J_i$. The summed private envelopes and sharing imply

$$d_0 \mu_i = y_i - h', \qquad J_i = \xi \mu_i, \qquad \delta_i = (1-\xi)\mu_i. \tag{B.6}$$

Entry, search, amenity choices and stocks therefore agree with the planner system. To establish sufficiency, let $\bar{x}$ be this stationary allocation, $\bar{\mu} > 0$ its constant costates and $F(x)$ the next employment vector. Define the stationary Hamiltonian expression

$$\mathcal{H}(x) = r(x) + \beta \bar{\mu} \cdot F(x) - \bar{\mu} \cdot n. \tag{B.7}$$

Without OTJ each transition is $(1-\rho)n_i + \phi v_i^{\xi} (c_i s)^{1-\xi}$ with fixed $c_B = 1-\alpha, c_G = \alpha$. Concavity of matching, positive costates, concave home and amenity utility, and linear remaining terms make $\mathcal{H}$ concave. Its full gradient, including employment coordinates, vanishes at $\bar{x}$. Thus $\mathcal{H}(x) \leq \mathcal{H}(\bar{x})$.

For any feasible alternative path with the common initial stock $n_0 = \bar{n}$, discounted

summation gives

$$\sum_{k=0}^{T} \beta^k [r(x_k) - r(\bar{x})] \leq -\beta^{T+1} \bar{\mu} \cdot (n_{T+1} - \bar{n}). \tag{B.8}$$

Employment is bounded and $\beta < 1$, so the right side tends to zero. The stationary allocation is optimal from those initial stocks. The inequality is obtained on the larger positive domain and therefore also holds on its subset satisfying all monthly probability bounds. It does not assert uniqueness or describe optimal policy from arbitrary initial stocks.

B.3 Positive-tax restriction

For a stationary private equilibrium at tax τ, let $\bar{L}$ and $\bar{A}_i$ be its participation and amenity totals. Constrain the planner in every month to

$$s_k + n_{B,k} + n_{G,k} = \bar{L}, \qquad A_{i,k} = \bar{A}_i. \tag{B.9}$$

Search, vacancies and employment composition can still adjust. With multiplier χ_L on $\bar{L} - s - n_B - n_G$, $\kappa = h' + \chi_L$. Set

$$\mu_i = \delta_i/t + J_i, \qquad \kappa = h'/t, \qquad \chi_L = \tau\kappa, \qquad \chi_{A_i} = -\tau. \tag{B.10}$$

The no-OTJ private allocation then satisfies the constrained planner conditions. The multipliers add linear terms to the Hamiltonian, leaving concavity intact; their contributions vanish along every path satisfying (B.9). The same telescoping argument establishes conditional optimality.

Fixing aggregate amenity expenditure alone is equivalent here. Conditional on its total $\bar{A}$, the planner allocates services as $A_i = \zeta_i^a \bar{A} / \sum_j \zeta_j^a$, reproducing private totals when $\bar{A} = \sum_j \zeta_j^a / t$. All amenity multipliers are equal, so one aggregate constraint suffices. Without the participation and amenity restrictions, private positive-tax choices do not equal the resource optimum: in particular private $A_i = \zeta_i^a / t$ differs from unrestricted $A_i = \zeta_i^a$.

B.4 OTJ implementation and a feasible improvement

At a candidate shared zero-tax private/planner allocation, the upper envelopes imply $\mu_G = \delta_G + J_G$. Planner upper entry requires $J_G = \xi(\mu_G - p\mu_B)$. Comparing private and planner search, with a positive lower channel, then requires $\delta_B = (1-\xi)\mu_B$, while lower entry requires $J_B = \xi\mu_B$. These statements imply (3.8); the summed lower envelope is consistent because $\delta_G - \mu_B = \mathcal{A}$. If $0 < p\mu_B < \mu_G$, the resulting rung-specific shares lie strictly between zero and one. Feasibility of cash compensation and acceptance must still be checked.

In the benchmark calibration's zero-tax resource-planner candidate, the upper employment share is .217060, compared with .268403 privately, and upper tightness is .574293, compared with .961635. Weights $(.5, .6115790832)$ implement that stationary candidate at zero tax when substituted into the original private system. The maximum allocation discrepancy is 3.56×10^{-15}. The OTJ planner Hamiltonian need not retain the no-OTJ concavity property, so these first-order conditions do not establish global optimality or implementation away from the reported candidate.

An independent feasible deviation establishes inefficiency from common initial stocks. Starting at the private steady state, reduce upper vacancies by 10 percent only in month zero, retaining current search and amenity totals. Subsequently follow

$$s_k = \bar{L} - n_{B,k} - n_{G,k}, \quad v_{B,k} = \bar{\theta}_B(1-\alpha)s_k, \quad v_{G,k} = \bar{\theta}_G(on_{B,k} + \alpha s_k), \quad A_{i,k} = \bar{A}_i. \tag{B.11}$$

Employment evolves according to the matching laws. This rule preserves participation and total amenities while keeping future tightness at its original level.

Under the rule, employment follows $n' = Bn + d$ and current welfare is affine in employment. If its gradient is g_r, the exact continuation gradient is $(I - \beta B^\top)^{-1} g_r$. The gain equals the initial vacancy resource saving plus the discounted value of the resulting change in next-period employment. A 601-month direct simulation with the exact remaining tail verifies the calculation. Search, employment and consumption remain positive, all monthly probabilities

Table A1: A feasible upper-vacancy reduction from common initial stocks

Calibration	Zero-tax gain	Baseline-tax gain
Benchmark, OTJ on	0.000670993	0.000601335
Benchmark, OTJ off	-0.000042600	-0.000038146
Rounded targets, OTJ on	0.000667621	0.000598313
Rounded targets, OTJ off	-0.000042989	-0.000038494

Discounted utility gains in final-good units under (B.11). The intervention is a one-month 10 percent reduction in upper vacancies. These are feasible gains, not optimal-policy gains or marginal-tax DWL.

are feasible, and the continuation matrix is stable.

The positive OTJ entries establish an additional inefficiency even with participation and amenities fixed. The negative no-OTJ entries are consistent with the benchmark sufficiency result. The feasible deviation establishes the gain directly rather than relying on planner first-order conditions alone.

C Calibration formulas and scale equivalence

C.1 Flow closure and matching

In calibration C, at fixed $\beta, \tau, \xi, \psi, \nu$, the eleven free primitives are

$$(\rho, \phi, \alpha, o, y_B, y_G, \gamma_B, \gamma_G, \zeta, \zeta_B^a, \zeta_G^a).$$

The eight target restrictions are $L, u, f_U, V/s, A/Z, r, \eta, m$. Output normalization, equal baseline amenities and an exposure normalization complete the count. In the baseline numerical Jacobian, the ten restrictions excluding exposure normalization have rank ten, while adding $\log o$ gives rank eleven.

To derive (4.7)–(4.10), stationarity of total employment gives $\rho N = f_U s$. Upper inflows give $\rho n_G = \lambda n_B + \alpha f_G s$, and the share from nonemployment is $\eta = \alpha f_G s/(\rho n_G)$. Consequently $rN = (1-\eta)\rho n_G$, yielding $g = r/[\rho(1-\eta)]$. Interior feasibility requires $0 < g < 1$. Then set

$n_B = (1-g)N$, $n_G = gN$ and use (4.9) and (4.10) for λ and c_α.

For any proposed $o > 0$, define

$$\alpha = c_\alpha o, \quad f_G = \lambda/o, \quad F_G = \alpha f_G = c_\alpha \lambda, \quad F_B = f_U - F_G, \quad f_B = F_B/(1-\alpha). \tag{C.1}$$

Require positive pools and rates and $0 < \alpha < 1$. If $T_v = (v_B + v_G)/s$, the unique positive common matching efficiency is

$$\phi = \left[\frac{(1-\alpha) f_B^{1/\xi} + (S_G/s) f_G^{1/\xi}}{T_v} \right]^{\xi}. \tag{C.2}$$

Recover $\theta_i = (f_i/\phi)^{1/\xi}$, $v_i = \theta_i S_i$ and $q_i = f_i/\theta_i$. These formulas fit aggregate vacancies and let market tightness be determined within the model.

C.2 Compensation and preference calibration

Set temporary $w_B^* = 1, w_G^* = x = \exp(m)$ and use (6.2), with $k = 1$ in the baseline. The temporary firm values solve

$$\begin{pmatrix} d_B + \beta F_B & \beta(F_G - \lambda) \\ \beta F_B & d_0 + \beta F_G \end{pmatrix} \begin{pmatrix} J_B^* \\ J_G^* \end{pmatrix} = \frac{1}{b} \begin{pmatrix} 1 + a_B^* \\ x + a_G^* \end{pmatrix}. \tag{C.3}$$

The determinant is $d_B(d_0 + \beta f_U) > 0$. Set $y_B^* = 1 + a_B^* + d_B J_B^*$ and $y_G^* = x + a_G^* + d_0 J_G^*$. The positive common scale $Y/(y_B^* n_B + y_G^* n_G)$ multiplies wages, amenities, firm values and productivities. Finally set

$$\gamma_i = \beta q_i J_i, \quad \delta_i = tbJ_i, \quad \zeta = \beta(F_B \delta_B + F_G \delta_G)(1-L)^\nu, \quad \zeta_i^a = t a_i n_i. \tag{C.4}$$

Algebraic invertibility does not ensure positive values or admissible compensation. The full equilibrium, resources, strict ranking and all monthly probabilities are checked independently.

Table A2: Calibrated primitives

Parameter	Benchmark	Rounded targets
Separation ρ	0.02700000	0.02647558
Matching efficiency ϕ	0.55378225	0.55486821
Upper exposure α	0.09605785	0.09720260
OTJ scale o	0.01000000	0.01000000
Home-utility scale ζ	0.45241875	0.45274828
Productivity B	1.63748987	1.63427898
Productivity G	1.92303308	1.91899558
Vacancy cost B	2.10542501	2.10781883
Vacancy cost G	4.14216683	4.15207856
Amenity taste B	0.05407738	0.05372722
Amenity taste G	0.01983954	0.02024864

Fixed parameters are $\beta = .996$, $\tau = .189$, $\xi = .5$, $\psi_B = \psi_G = .5$, and $\nu = 1$. Production is linear, amenity resource cost is one, and consumption is quasilinear. o is a disclosed normalization; all tax experiments hold these primitives fixed.

For general k, strict ranking is equivalent here to $x + a_G^* > 1 + a_B^*$. Writing $\omega = A/Z$, indifference occurs at

$$k_* = \frac{\omega(1 - g + xg) - (x - 1)(1 - g)}{\omega(1 - g + xg) + (x - 1)g}. \tag{C.5}$$

The benchmark and rounded-target boundaries are .186140806 and .190030889. The sensitivity code checks points .01 and .001 above each boundary. It records the declared $k = .10$ as inadmissible rather than silently altering the wage target or acceptance rule.

C.3 Why exposure scale is not separately identified

Define recruiting expenditure $E_i = \gamma_i v_i$. Substituting free entry into the matching coefficients gives

$$\begin{aligned} b_B &= \phi \gamma_B^{-\xi} (1 - \alpha)^{1-\xi} = \frac{F_B^{1-\xi}}{(\beta J_B)^{\xi}}, \\ b_G &= \phi \gamma_G^{-\xi} o^{1-\xi} = \frac{\lambda^{1-\xi}}{(\beta J_G)^{\xi}}. \end{aligned} \tag{C.6}$$

Table A3: Baseline allocation and compensation

Object	Benchmark	Rounded targets
s	0.0366139	0.0359600
n_B	0.4268034	0.4241768
n_G	0.1565827	0.1598632
g	0.2684032	0.2737196
v_B	0.0188756	0.0183711
v_G	0.0074864	0.0075201
θ_B	0.5703139	0.5658798
θ_G	0.9616352	0.9719495
w_B	1.3766148	1.3753327
w_G	1.5393027	1.5378691
a_B	0.1562309	0.1561804
a_G	0.1562309	0.1561804
J_B	2.8826929	2.8691034
J_G	7.3643496	7.4069297
δ_B	2.3378639	2.3268428
δ_G	5.9724876	6.0070200
Wage bill Z	0.8285721	0.8292329
Amenities A	0.0911429	0.0912156
Recruitment K	0.0707511	0.0699471
Dividends Π	0.0095339	0.0096044
Consumption c	0.8381060	0.8388373

Masses are population shares; pay and spending use final-good units. Output equals one and participation equals .62. Rungs are ranked by worker values.

Table A4: Monthly probability diagnostics

Probability	Benchmark	Rounded targets
f_B	0.4182115	0.4173995
f_G	0.5430555	0.5470307
q_B	0.7333006	0.7376116
q_G	0.5647209	0.5628180
f_U	0.4302038	0.4300000
λ	0.0054306	0.0054703
$\rho + \lambda$	0.0324306	0.0319459

All reported equilibria satisfy the effective-unit finding and filling bounds, the nonemployed finding bound, and the competing-exit bound.

The right sides, c_α, productivities and preference scales do not depend on the chosen admissible positive exposure normalization. Equations (4.12)–(4.13) thus describe the same matching technology in expenditure units. Resources cost $E_B + E_G$ and the employed-origin share is $n_B/(n_B + c_\alpha s)$, also unchanged. The private and planner real conditions coincide under this transformation.

Within each calibrated economy, setting only o to zero changes the upper expenditure-space search term from $n_B + c_\alpha s$ to $c_\alpha s$, with the same coefficient b_G. Thus the corresponding on/off comparisons also agree across calibration normalizations. This is not a comparison in which o changes and the other physical-unit primitives are all held fixed across calibrations.

The equivalence holds on the common admissible domain. Effective-unit f_i and q_i bounds can exclude different remote allocations under different normalizations, so global equality of capped feasible sets is not asserted. Physical vacancy counts and tightness need not agree away from the fitted baseline. Under benchmark targets, admissible baseline o lies between .005430555 and .018818958; the exact-target interval is [.005470307, .018932761]. The lower and upper endpoints are bound by $f_G = 1$ and $q_G = 1$.

The strict nonwage proxy has a smaller upper bound and uses $o = .00562250$ under benchmark targets, rather than .01. Some bootstrap draws require an additional change of normalization to fit inside both the on and off probability domains. For $\xi = .5$, free entry implies $q_{i,0}/q_{i,1} = J_{i,1}/J_{i,0}$ and $f_{i,0}/f_{i,1} = J_{i,0}/J_{i,1}$. Intersecting the implied finding and filling caps gives the common interval; its midpoint is used when necessary. This occurs in 11 inherited and 9 exact-target admitted strict-nonwage draws. The calibration moments are not changed, and each resulting on/off pair still differs only in o.

D Identical markets and the controlled wage ladder

D.1 Nesting the one-type economy

Take a one-type stationary equilibrium with $o = 0$, employment N, vacancies V, nonemployed search s, amenity taste ζ_a and per-worker allocation (w, a, δ, J). For any $0 < \alpha < 1$, let $p_B = 1 - \alpha$ and $p_G = \alpha$. Duplicate its productivity, recruiting cost, matching technology and bargaining weight in both markets, and set

$$n_i = p_i N, \quad v_i = p_i V, \quad S_i = p_i s, \quad \zeta_i^a = p_i \zeta_a. \tag{D.1}$$

Constant returns give the original tightness, finding and filling rates in each market. The stock equations and free-entry conditions are therefore satisfied. The amenity condition gives $a_i = \zeta_i^a/(tn_i) = a$; both worker envelopes and bargaining conditions coincide with the one-type conditions. The search condition sums identical opportunities with weights $p_B + p_G = 1$. Resources, participation, total amenities and taxable pay therefore coincide as well.

At any nearby tax on the same interior branch, $A_i = p_i A$. The only difference in stationary flow utility is

$$\sum_i \zeta_i^a \log A_i - \zeta_a \log A = \zeta_a \sum_i p_i \log p_i. \tag{D.2}$$

This is constant in the tax, so all aggregate stationary derivatives and welfare metrics coincide exactly. The construction changes the division of amenity tastes consistently with recruitment. Varying α while keeping the two tastes individually fixed would be a different experiment. Equal job values are admissible here because $o = 0$: there is no positive climbing flow requiring strict acceptance. Table A5 reports three recruitment shares and the corresponding utility constants.

Table A5: Identical-market nesting at different recruitment shares

α	R	H	Utility constant	Aggregate discrepancy
0.096058	1.071822385	0.166640238	-0.023821598	1.33×10^{-15}
0.300000	1.071822385	0.166640238	-0.046001289	5.11×10^{-15}
0.500000	1.071822385	0.166640238	-0.052197621	6.66×10^{-16}

The amenity tastes and employment masses split in proportions $(1-\alpha, \alpha)$. The utility constant is the two-type minus one-type value in (D.2). It has no effect on the tax derivative or the welfare comparison.

D.2 The equal-tightness calibration and aggregation result

Fix $(\beta, \tau, \xi, \psi, \nu, \alpha, o = 0)$ and the common targets $(L, u, f_U, V/s, Y, \omega)$, where $\omega = A/Z$. Write $N = (1-u)L$, $s = uL$, $\theta = V/s$, $f = f_U$, $q = f/\theta$, $d_0 = 1 - \beta(1-\rho)$ and $b = \psi/(1-\psi)$. Equal tightness and common matching efficiency imply

$$\rho = \frac{fs}{N}, \quad \phi = \frac{f}{\theta^{\xi}}, \quad n_i = p_i N, \quad v_i = p_i \theta s. \tag{D.3}$$

These objects are independent of the wage premium m.

Set $Q = h'/t$ and $e_i = w_i + a_i$, and let $\overline{J} = \sum_i p_i J_i$ and $\overline{e} = \sum_i p_i e_i$. The worker and firm envelopes give

$$e_i = b d_0 J_i + Q, \qquad y_i = e_i + d_0 J_i, \qquad Q = \beta f b \overline{J}. \tag{D.4}$$

Aggregating production fixes the average firm value:

$$\overline{J} = \frac{Y}{N[(1+b)d_0 + b\beta f]}, \qquad \overline{e} = b(d_0 + \beta f)\overline{J}. \tag{D.5}$$

Equal per-worker amenities and the expenditure target imply

$$\overline{w} = \frac{\overline{e}}{1+\omega}, \qquad a = \omega \overline{w}. \tag{D.6}$$

For $x = \exp(m)$, the unique wage pair with that mean and relative wage is

$$w_B = \frac{\overline{w}}{1 - \alpha + \alpha x}, \qquad w_G = x w_B. \tag{D.7}$$

Recover each market's remaining primitives and values as

$$\begin{aligned} J_i &= \frac{w_i + a - Q}{b d_0}, & y_i &= w_i + a + d_0 J_i, \\ \gamma_i &= \beta q J_i, & \delta_i &= t b J_i, \\ \zeta &= tQ(1-L)^{\nu}, & \zeta_i^a &= t a n_i. \end{aligned} \tag{D.8}$$

The construction is admissible whenever these values are positive and the common matching probabilities lie in their admissible domain.

We now prove Proposition 4, stated in Section 5: along the admissible equal-tightness calibration in (D.3)–(D.8), the aggregate stationary allocation and its local tax derivatives are independent of m, so the ETI, the ETI prediction, the welfare correction Ω and loss in goods H are identical to those at $m = 0$.

Proof. The formulas fix L, s, N, total vacancies, Y and mean effective compensation independently of m. They also fix

$$Z = N\overline{w}, \quad A = Na, \quad K = \beta f s \overline{J}, \quad \Pi = (1-\beta) N \overline{J}. \tag{D.9}$$

The home-utility and amenity preference scales in (D.8) do not change. On the regular local tax branch, matching conditions and firm values are constant in each calibrated economy. Hence the common aggregate responses are

$$\begin{aligned} L_\tau &= -\frac{1-L}{\nu t}, & A_\tau &= \frac{A}{t}, \\ Z_\tau &= (Z + A)\frac{L_\tau}{L} - \frac{A}{t}, & (Y_\tau, K_\tau, \Pi_\tau) &= (Y, K, \Pi)\frac{L_\tau}{L}. \end{aligned} \tag{D.10}$$

Proposition 3 then gives the same H, D_ε and R. The equality holds analytically; the numerical comparisons below verify implementation of this construction in the original equilibrium system. □

This result concerns the stated calibration construction. Equal tightness is an additional restriction, and the taste scales, productivity and recruiting costs are calibrated as above. It does not imply that arbitrary changes in wage dispersion or arbitrary heterogeneous no-OTJ economies preserve aggregate welfare accounting. Individual wage and match-value responses need not coincide across the markets.

D.3 Parameter count, local identification and feasibility

The nine free primitives in B are

$$(\rho, \phi, y_B, y_G, \gamma_B, \gamma_G, \zeta, \zeta_B^a, \zeta_G^a).$$

Six common aggregate targets, the wage premium and equal amenities supply eight restrictions. Equal tightness supplies the ninth. Combining them with the thirteen original equilibrium equations gives 22 equations in 22 unknowns. Without the closure there are 21 equations in those same 22 unknowns. No welfare metric is included among the restrictions.

The numerical implementation starts with the symmetric equilibrium and continues m from zero to .111702059 at 41 equally spaced points. It solves the original equilibrium and calibration residuals before evaluating the closed-form solution as an independent check. At every point, the joint Jacobian has row rank 21 without the closure and rank 22 with it, using a relative singular-value threshold of 10^{-10}. The reported condition numbers refer to the implementation's transformed variables and scaled residuals; they are not invariant to numerical units. Table A6 reports selected points.

Feasibility between the reported points also follows from the explicit solution. For $m \geq 0$, w_B and J_B decrease, while w_G and J_G increase. The final $J_B = 2.970567$ remains positive.

Table A6: Selected points on the equal-tightness continuation

m	J_B	J_G	Rank, open/closed	Condition number
0.000000	3.496321	3.496321	21/22	120.753
0.027926	3.369251	4.692096	21/22	93.105
0.055851	3.239300	5.914988	21/22	77.076
0.083777	3.106421	7.165435	21/22	67.578
0.111702	2.970567	8.443875	21/22	64.307

The complete path has 41 equally spaced wage premiums. All points pass the original equilibrium, calibration and feasibility checks. The open system has 21 equations and 22 unknowns; equal tightness makes the closed system square. Condition numbers use transformed coordinates and scaled residuals.

The amenity level, employment shares and matching probabilities do not change, so positivity and probability feasibility hold throughout the interval. For $m > 0$, $w_G > w_B$ and equal amenities imply strict worker-value ranking. At $m = 0$, the identical-market endpoint remains a valid no-OTJ equilibrium.

Three perturbed endpoint starts, with log-coordinate standard deviation .08 and seeds 613, 947 and 2026, recover the same solution. A separate generic equilibrium start also agrees. These checks and the Jacobian ranks establish local identification and continuation of the reported branch; they do not establish global uniqueness of all heterogeneous calibrations. Table A7 reports the numerical discrepancies, including implicit derivatives and central differences at tax steps 10^{-4}, 10^{-5} and 10^{-6}.

D.4 Complete A/B/C calibration and allocations

Tables A8 and A9 give the primitives and allocations behind the common-target sequence. In A and B, $g = \alpha$ because finding probabilities are equal and only nonemployed workers enter the two markets. In C, upward mobility changes the upper employment share even though the direct-exposure share is held fixed. The structural OFF counterpart retains C's primitives except for o; its detailed allocation is included in Table A9.

Table A7: Nesting, continuation and derivative verification

Check	Maximum absolute discrepancy
Identical-market aggregate nesting	5.11×10^{-15}
Identical-market derivative nesting	1.33×10^{-15}
Log-utility constant identity	5.27×10^{-16}
B continuation: equation residual	4.11×10^{-15}
B continuation: closed-form coordinates	3.29×10^{-14}
B continuation: aggregates relative to A	4.44×10^{-15}
B continuation: derivatives relative to A	2.33×10^{-15}
B endpoint: perturbed-start discrepancy	9.55×10^{-15}
All benchmark central-difference metrics	1.21×10^{-8}

Identical-market checks use $\alpha = 0.09605785$, .3 and .5 with consistently split amenity tastes. Central differences independently solve nearby-tax equilibria at steps 10^{-4}, 10^{-5} and 10^{-6}. The acceptance tolerances are 10^{-9} for equations and aggregate nesting, 2×10^{-9} for closed-form coordinates, and 2×10^{-7} for derivative metrics.

Table A8: Calibrated primitives in the A/B/C sequence

Primitive	A	B	C
ρ	0.027000000	0.027000000	0.027000000
ϕ	0.507000000	0.507000000	0.553782245
o	0.000000000	0.000000000	0.010000000
y_B	1.714130576	1.681647372	1.637489869
y_G	1.714130576	2.019810280	1.923033077
γ_B	2.080713731	1.767829354	2.105425011
γ_G	2.080713731	5.025078632	4.142166825
ζ	0.461688738	0.461688738	0.452418749
ζ_B^a	0.068071589	0.068071589	0.054077381
ζ_G^a	0.007233661	0.007233661	0.019839536

All cases share $\beta = .996$, $\tau = .189$, $\xi = \psi = .5$, $\nu = 1$ and $\alpha = 0.09605785$. The structural OFF counterpart uses the same primitives with only o set to zero. Values are computed at full precision; displayed rounding is not used as an input to the equilibrium solver.

Table A9: Allocations and resource accounts across benchmarks

Object	A	B	C	C with OTJ off
L	0.6200000	0.6200000	0.6200000	0.6181593
s	0.0366139	0.0366139	0.0366139	0.0367079
n_B	0.5273473	0.5273473	0.4268034	0.5094277
n_G	0.0560388	0.0560388	0.1565827	0.0720237
v_B	0.0238297	0.0238297	0.0188756	0.0185915
v_G	0.0025323	0.0025323	0.0074864	0.0034971
θ_B	0.7200000	0.7200000	0.5703139	0.5602929
θ_G	0.7200000	0.7200000	0.9616352	0.9917816
w_B	1.4469570	1.4307154	1.3766148	1.4183318
w_G	1.4469570	1.5997968	1.5393027	1.3523422
a_B	0.1591653	0.1591653	0.1562309	0.1308917
a_G	0.1591653	0.1591653	0.1562309	0.3396529
J_B	3.4963209	2.9705666	2.8826929	2.8572548
J_G	3.4963209	8.4438753	7.3643496	7.4788916
Y	1.0000000	1.0000000	1.0000000	0.9726867
Z	0.8441346	0.8441346	0.8285721	0.8199382
A	0.0928548	0.0928548	0.0911429	0.0911429
K	0.0548517	0.0548517	0.0707511	0.0536286
Π	0.0081588	0.0081588	0.0095339	0.0079769

A/B/C share normalized output $Y = 1$. The final column retains the goods units of C. The full stationary derivatives and additional matching probabilities are supplied in the replication results.

E Compensation and resource identification

E.1 Redistribution at fixed total amenity taste

Hold the model's primitives fixed except for the allocation of amenity tastes. Fix $S_a = \zeta_B^a + \zeta_G^a > 0$ and set

$$\zeta_B^a = (1-\sigma)S_a, \qquad \zeta_G^a = \sigma S_a, \qquad 0 < \sigma < 1. \tag{E.1}$$

This is a change in preferences across economies, distinct from changing the tax within an economy and from recalibrating latent primitives to preserve the declared observations.

Proposition 5 (Fixed-total-taste invariance)**.** *On a common admissible regular branch,*

redistribution in (E.1) *leaves matching, employment, participation, aggregate resource accounts and their tax derivatives unchanged. The result holds with and without OTJ. It preserves ETI, R and H, but need not preserve utility levels across taste specifications.*

Proof. The reduced system (3.9)–(3.12) contains neither taste weight. Thus tightness, J_i, Q, e_i and all contact rates are unchanged. The home-time condition $h'(1-L) = tQ$ fixes participation independently of the taste split. The stock equations then fix employment and vacancies. The amenity and compensation conditions give

$$A = \frac{S_a}{t}, \qquad a_B = \frac{(1-\sigma)S_a}{tn_B}, \qquad a_G = \frac{\sigma S_a}{tn_G}, \qquad w_i = e_i - a_i. \tag{E.2}$$

Therefore $Z = \sum_i e_i n_i - S_a/t$ is unchanged, as are Y, K and Π. These relations hold for every nearby tax in the common domain, so their tax derivatives coincide. In the amenity utility term,

$$\sum_i \zeta_i^a \log A_i = S_a \log(S_a/t) + S_a[(1-\sigma)\log(1-\sigma) + \sigma \log \sigma]. \tag{E.3}$$

The last term is independent of the tax. Thus DWL, the ETI-based prediction and their ratio also coincide. □

The cash-pay constraints make the domain explicit. With $A = S_a/t$, $w_B, w_G > 0$ require

$$\max\left\{0, 1 - \frac{e_B n_B}{A}\right\} < \sigma < \min\left\{1, \frac{e_G n_G}{A}\right\}. \tag{E.4}$$

All other maintained positivity, ranking and probability restrictions are invariant. For C and its structural OFF counterpart, the unclipped bounds in Table A10 contain $(0, 1)$. The endpoints are excluded because the maintained specification has positive taste weights and positive amenities on both active rungs. Redistribution can reverse the cash-wage ranking without reversing the worker-value ranking that governs climbing. One hundred interior taste shares in each economy verify the analytical aggregate invariance.

Table A10: Admissible fixed-total-taste redistributions

Economy	Unclipped lower	Unclipped upper	Aggregate discrepancy
C: OTJ on	-6.177998	2.912911	1.84×10^{-13}
C with OTJ off	-7.659119	1.337062	5.33×10^{-14}

Bounds intersect with $0 < \sigma < 1$, so every interior share is admissible in these two economies. The last column is the maximum aggregate level or stationary welfare-metric discrepancy across the 100 evaluated shares in each economy.

E.2 Target-preserving latent calibrations

The latent calibration profile instead fixes the common aggregate targets, the relative cash wage $x = \exp(m)$, climbing and the nonemployed-origin share of upper hires, while imposing a selected $k = a_G/a_B$. The flow and matching formulas in Appendix C leave (n_B, n_G), contact rates and α fixed. In temporary units $w_B^* = 1, w_G^* = x$, set

$$\mathcal{Z} = n_B + x n_G, \qquad \mathcal{A} = \omega \mathcal{Z}, \qquad \chi(k) = \frac{k n_G}{n_B + k n_G}, \quad \omega = A/Z. \tag{E.5}$$

Then $a_B^* = \mathcal{A}(1-\chi)/n_B$ and $a_G^* = \mathcal{A}\chi/n_G$. These are the same amenity levels as (6.2). Solving (C.3) and applying the output normalization determines the entire parameter vector. Equal L, ω and ν imply equal ETI by (3.17); there is no additional ETI restriction in this calibration.

The profile therefore preserves the declared observations, including the amenity-to-wage expenditure ratio, rather than absolute expenditure or the taxable wage bill. Both can change in output units. Each structural OFF economy changes only o within its calibrated pair. Its employment, wages, amenity ratio, ETI and output are predictions, with no output renormalization after OTJ is removed. Table A11 records the separate compensation and exposure constructions.

Tables A12 and A13 give the complete exposure grids. Changing actual o alone changes productive opportunities and the equilibrium allocation. Recalibrating positive exposure units according to (C.6) preserves the real allocation, including the comparison obtained by

Table A11: Compensation and exposure experiments

Experiment	Construction	R	H
1. OTJ off	Only $o = 0$ in C	1.072201	0.163397
2. Off: taste redistribution	Fixed total taste; other primitives fixed	1.072201	0.163397
3. Off: paired latent calibrations	Only $o = 0$ within each ON fit	1.071480–1.076312	0.161016–0.165052
4. OTJ on	Baseline C	1.085504	0.165656
5. Actual OTJ opportunities	Vary o alone from 0 to twice baseline	1.072201–1.094112	0.163397–0.167121
6. Positive exposure units	Refit units with common targets	1.085504	0.165656
7. On: taste redistribution	Fixed total taste; other primitives fixed	1.085504	0.165656
8. On: latent calibrations	Preserve targets; vary k on [.25, 4]	1.073166–1.107337	0.165341–0.166199

All objects are stationary. Rows 2 and 7 redistribute amenity taste weights at a fixed sum and leave aggregate outcomes invariant. Rows 3 and 8 are paired latent calibrations: the observations are held fixed in the ON economy, not its OFF counterpart. Row 5 changes actual opportunities; row 6 changes exposure units. Ranges summarize the admitted reported grids.

Table A12: Changing OTJ exposure with all other primitives fixed

o	L	r	R	H	ε
0.000000	0.618159	0.000000	1.072201	0.163397	0.797527
0.002500	0.618661	0.001146	1.076149	0.164067	0.795550
0.005000	0.619133	0.002179	1.079636	0.164659	0.793699
0.007500	0.619579	0.003117	1.082735	0.165185	0.791960
0.010000	0.620000	0.003973	1.085504	0.165656	0.790323
0.012500	0.620399	0.004757	1.087990	0.166079	0.788777
0.015000	0.620778	0.005478	1.090233	0.166461	0.787316
0.020000	0.621482	0.006761	1.094112	0.167121	0.784616

Each row changes only o relative to baseline C. Participation, climbing and ETI adjust endogenously. Goods units remain those of C. The $o = 0$ row is the structural OFF economy; these rows do not preserve the calibration targets.

setting only $o = 0$ within each pair.

Strict climbing acceptance imposes the boundary derived in Appendix C. On the reported profile $k \in [.25, 4]$, all original equilibrium, cash-pay, ranking and probability checks pass. The rejected point $k = .10$ is not part of the reported welfare range. The attained correction

Table A13: Equivalent positive exposure normalizations

o	α	ϕ	γ_B	γ_G	Maximum real discrepancy
0.007500	0.072043	0.574476	2.325907	3.343139	2.13×10^{-14}
0.010000	0.096058	0.553782	2.105425	4.142167	1.60×10^{-14}
0.013828	0.132829	0.540256	1.922316	5.451413	8.35×10^{-14}

These calibrations preserve the common targets and real allocation by changing effective units as in (C.6). The last column is the largest reported real-allocation or welfare discrepancy relative to baseline C and its paired OFF economy. Zero exposure cannot reproduce positive climbing.

at $k = 4$ is not a global upper bound or a statistical confidence limit. Table A19 gives the full declared amenity grid.

E.3 Derivation and inversion of the accounting relationship

Let M denote the matrix on the left of (C.3), and define

$$\boldsymbol{e}_0 = \begin{pmatrix} 1 + \mathcal{A}/n_B \\ x \end{pmatrix}, \qquad \boldsymbol{e}_1 = \begin{pmatrix} -\mathcal{A}/n_B \\ \mathcal{A}/n_G \end{pmatrix}, \qquad \boldsymbol{J}_j = \frac{1}{b} M^{-1} \boldsymbol{e}_j \quad (j = 0, 1). \tag{E.6}$$

Thus $\boldsymbol{J}^*(k) = \boldsymbol{J}_0 + \chi(k)\boldsymbol{J}_1$. Temporary output is

$$\begin{aligned} \mathcal{Y}(k) &= \mathcal{Z} + \mathcal{A} + d_B n_B J_B^*(k) + d_0 n_G J_G^*(k) = \mathcal{Y}_0 + \mathcal{Y}_1 \chi(k), \\ \mathcal{Y}_0 &= \mathcal{Z} + \mathcal{A} + d_B n_B J_{0B} + d_0 n_G J_{0G}, \\ \mathcal{Y}_1 &= d_B n_B J_{1B} + d_0 n_G J_{1G}. \end{aligned} \tag{E.7}$$

The output normalization multiplies wages and output by the same factor. Their ratio therefore equals

$$z(k) \equiv \frac{Z(k)}{Y(k)} = \frac{\mathcal{Z}}{\mathcal{Y}_0 + \mathcal{Y}_1 \chi(k)}, \qquad z'(k) = -\frac{\mathcal{Z}\mathcal{Y}_1 n_B n_G}{[\mathcal{Y}_0 + \mathcal{Y}_1 \chi(k)]^2 (n_B + k n_G)^2}. \tag{E.8}$$

Direct evaluation of the calibrated matrix gives

$$\mathcal{Z} = 0.601891018, \quad \mathcal{Y}_0 = 0.716394748, \quad \mathcal{Y}_1 = 0.037349985 > 0. \tag{E.9}$$

Hence $z(k)$ is strictly decreasing on the admissible studied profile. This sign is established for the maintained calibration; it is not imposed as a general restriction on every model parameterization.

For an exact compatible ON observation z^{obs}, the inverse is

$$\chi^{\text{obs}} = \frac{\mathcal{Z}/z^{\text{obs}} - \mathcal{Y}_0}{\mathcal{Y}_1}, \qquad k^{\text{obs}} = \frac{n_B \chi^{\text{obs}}}{n_G(1-\chi^{\text{obs}})}. \tag{E.10}$$

The selected k must belong to the declared admissible profile; an observation outside its image rejects that profile. If k has already been fixed independently, z^{obs} instead supplies an overidentifying restriction. The analytical level and inverse formulas are checked against 751 independently calibrated points. The paired OFF relationship is computed by resolving the no-OTJ equilibrium, and its monotonicity is verified numerically on the same interval.

E.4 Illustrative precision and the measurement boundary

For an interval $z^{\text{obs}} \in [z_-, z_+]$ observed in the ON economy, strict monotonicity gives the interval of compatible k values by inverting the two bounds in reverse order and intersecting with $[.25, 4]$. An empty intersection rejects the studied profile. Table A14 reports two illustrative half-widths around the baseline ON accounting ratio and, separately, around the baseline OFF ratio. The latter is a different information experiment: it presumes a compatible accounting observation of the OFF economy. Neither experiment supplies an empirical confidence interval.

Each interval of selected ON calibrations also selects its paired OFF predictions. Bounds on welfare objects use their extrema over the selected interval. In particular, R_{off} has an interior minimum near $k = .264532$, so endpoint values alone need not give every welfare

Table A14: Illustrative accounting precision and the selected calibration pairs

Observed economy	Half-width	k interval	$100(R_{\text{on}} - 1)$	$100(R_{\text{on}} - R_{\text{off}})$
ON	0.005	[0.4876, 1.7170]	[7.7698, 9.3404]	[0.6105, 2.0185]
ON	0.010	[0.2500, 2.7919]	[7.3166, 10.1401]	[0.1686, 2.6669]
OFF	0.005	[0.2500, 2.8966]	[7.3166, 10.2016]	[0.1686, 2.7141]
OFF	0.010	[0.2500, 4.0000]	[7.3166, 10.7337]	[0.1686, 3.1025]

Bands are centered at baseline ON $Z/Y = .828572$ or baseline OFF $Z/Y = .842962$, as indicated, and intersected with the declared $k \in [.25, 4]$ profile. They are hypothetical measurement-precision experiments, not estimated empirical bands. The fourth column is excess over the ETI prediction in percent; the last is the paired correction increment in percentage points.

bound. The machine-readable results retain that extremum and the full profiles. The accounting inverse is conditional on the declared one-dimensional calibration profile. It does not identify unrestricted compensation heterogeneity.

The empirical numerator must match taxable cash compensation, and the denominator must match resources available for $Z + A + K + \Pi$. Here K denotes recruiting expenditure, not a physical capital stock or capital income. A labor share formed from conventional gross value added can include a different compensation basket and resources assigned to capital services, depreciation or production taxes that are absent from the model. Sector and period coverage, the treatment of those charges, the distinction between cash pay and benefits, and the resource boundary for recruiting must be reconciled jointly. Wage-bill and output observations with incompatible boundaries cannot supply the additional moment in (E.10).

The analysis consequently reports attainable model ratios and the implications of specified measurement precision. It does not select a narrow empirical target by treating a conditional accounting scenario as an independent measurement. This distinction preserves the identification content of the exercise: the maintained labor-market observations leave latent resource information relevant for DWL.

F Survey construction and empirical qualifications

F.1 Sample and variable rules

The source is the archived SCE Labor Market Survey workbook, sheet `Data`, with headers on row two. It contains 36,561 observations; 20,710 have positive weights and dates in 2014–2019. Employment requires a valid routed `lmtype` classification from one to five. Nonemployment requires affirmative routing evidence through `l7y` or `l7dk_1`, without a valid employment classification. The resulting counts are 12,056 employed, 5,713 nonemployed and 2,941 unclassified. This is a routed wage/salary-employment sample, not a complete CPS-style labor-force classification.

We link a respondent only across adjacent observations exactly four months apart, obtaining 10,917 pairs from 6,940 respondents. Weights are the mean of the two positive cross-sectional weights. The 6,244 prior-employed exposure pairs include 188 subsequently nonemployed and 144 subsequently unclassified observations. Retaining them avoids restricting the exposure denominator to employment survivors.

A new spell requires a valid current job start month after the prior interview, no later than the current interview, and different from the previously reported start month. There are 369 classified new spells, 254 from employment and 115 from routed nonemployment, representing 357 respondents. An additional 39 new spells have unknown origin and are excluded from hire numerators.

Overall satisfaction uses `lmsat5`; the nonwage version uses `lmsat2`. The broad criterion assigns four and five to the upper group; the strict criterion assigns only five. Annual pretax earnings use `l3`, including bonuses and other payments. Pay observations must lie between 1,000 and 1,000,000, and the absolute log change must not exceed $\log 4$. There is no hours adjustment or fallback imputation from earnings bins.

The baseline's mean log gain uses 88 of its 98 upward-proxy moves. Approximately 32.70 percent of valid upward-proxy pairs have a cash earnings decline. The equal-amenity

Table A15: Alternative satisfaction proxies

Proxy	NE to high	Low to high	High to high	Pay pairs
Overall ≥ 4	74	98	84	88
Overall $= 5$	28	64	23	58
Nonwage ≥ 4	51	63	113	52
Nonwage $= 5$	15	48	21	38

Unweighted counts from adjacent four-month SCE observations, 2014–2019. Moment estimates use pair weights. High-to-high moves are excluded from the two-rung inflow mapping, not assumed to be absent from the data.

calibration fits the selected mean wage gain, not that individual distribution. Among the 74 nonemployed-to-high hires, 49 report previous search and 25 do not; the weighted search share is .588877. Even this searching subgroup does not directly observe availability for a full unemployment definition.

The New York Fed fields the labor survey every four months (New York Fed, 2026). Endpoint observations do not reveal every intervening employer, an uninterrupted employment history or exact time at risk. Dividing the upward count by four supplies the maintained monthly proxy. A constant-hazard conversion would give .003996875 rather than .003972977 in the baseline, but does not resolve missing intermediate histories.

F.2 Stocks, reclassification and other definitions

The four proxies produce high-satisfaction shares of .681036, .273514, .628545 and .247912. Their model-implied benchmark-target upper shares are .268403, .142991, .184311 and .098447. Among 4,221 linked observations with the same valid start date and valid endpoint ranks, the corresponding binary reclassification rates are .181949, .158036, .209223 and .183860. Neither this start-date proxy nor satisfaction itself directly observes a permanent job type. These statistics make the permanent-rung interpretation a substantive empirical assumption.

The exact-target aggregate convention changes model upper shares slightly while leaving these survey statistics unchanged. Alternative proxy results are sensitivity to definitions; they are not independent samples that resolve the mismatch.

Three further maintained-mapping qualifications underlie the estimates in (4.3)–(4.5). Four-month survey endpoints divided by four are not fully observed monthly hazards; nonemployment includes inactivity, and the annual earnings entering $\widehat{m}$ include variation in hours and other pay. The model admits no G-to-G moves, whereas the baseline data contain 84 high-to-high switches. These switches are excluded from the upper-hire denominator in (4.4), but remain in the previously employed exposure denominator in (4.3). Including them as upper hires would lower the nonemployed-origin share to .3090. These are interpretation limitations of the proxy, not adjustments to the calibrated targets.

F.3 Offer evidence and the units of exposure

The published Faberman et al. (2022) Table IV.B reports formal-offer incidence over four weeks of .173 for employed searchers, .051 for employed nonsearchers, .081 for all employed, .342 for unemployed and .079 for those outside the labor force. Their search-supplement sample and the labor-workbook panel used here are distinct. Offer incidence also differs from offers per application: the former employed/unemployed ratio is approximately .237, whereas the latter relative yield is approximately 3.22. Neither number is substituted for o.

In our model, accepted upper contacts per lower employee are of_G, accepted upper contacts per nonemployed searcher are αf_G, and finding any job is f_U. Even compatible upper-contact data identify o/α, rather than o alone. Broad employed offer statistics mix lower and upper employees, search participation and rejected offers. Rung-specific physical vacancies, recruiting costs in known units, or a compatible exposure allocation measure would supply information absent from that incidence comparison.

F.4 Joint bootstrap and reproducibility

The archived respondent-cluster bootstrap supplies 999 joint (r, η, m) draws for each definition. An independent grouped-sum reconstruction reproduces all 3,996 archived draws within 1.95×10^{-16}. The structural exercise maps each joint draw under both aggregate conventions

Table A16: Joint bootstrap ranges conditional on model admission

Proxy	Admitted	ΔR, pp	ΔH, %	ΔD, %
Overall ≥ 4	992/999	[0.384, 2.255]	[0.399, 2.344]	[-1.110, 1.693]
Overall $= 5$	999/999	[0.680, 2.686]	[0.706, 2.794]	[-1.123, 0.497]
Nonwage ≥ 4	994/999	[0.348, 2.474]	[0.361, 2.571]	[-1.203, 1.001]
Nonwage $= 5$	905/999	[0.060, 1.786]	[0.062, 1.857]	[-0.473, 0.994]

2.5th–97.5th percentiles of the admitted joint respondent draws. These are not unconditional or official survey-design confidence intervals. Worker-ranking rejections are 7, 0, 5, and 81; the final proxy has 13 additional positive-cash-pay failures in the off economy. All attempted draws are retained in the replication data.

and records every failure of ranking, cash-pay positivity or the probability domain.

For overall satisfaction at least four, the conditional range for R_{on} is [1.075322,1.096364] under benchmark targets. Conditioning is explicit because admission is an economically relevant restriction, especially under the strict nonwage proxy. Failed comparisons are retained with their input moments and reasons, not assigned zero effects or silently omitted from the attempted-draw count. These are proxy sampling distributions, not official design-based uncertainty estimates or a solution to structural identification.

Source: Survey of Consumer Expectations, Federal Reserve Bank of New York. The Bank is not responsible for our analysis or interpretation. The replication files retain the original data, questionnaire, variable mapping and source hashes.

G Additional results and numerical verification

G.1 Alternative aggregate targets and finite tax changes

The alternative calibration retains the common restrictions but imposes the reported rounded targets $u = .058$ and $f_U = .43$ exactly, rather than retaining the benchmark separation rate. Table A17 reports the three economies. Table A18 compares directly solved finite tax changes with the integrated welfare identity under the benchmark calibration.

The full-system equilibrium solver takes structural parameters and a tax rate, solving

Table A17: Alternative aggregate calibration

Economy	ε	D_ε	D	R	H
One-type ENOP	0.790323	0.184181	0.197430	1.071936	0.166819
Ladder, OTJ off	0.797876	0.185941	0.199394	1.072347	0.163542
Ladder, OTJ on	0.790323	0.184181	0.200033	1.086067	0.165874

Targets are $u = .058$ and $f_U = .43$ exactly; all other aggregate targets are unchanged.

Table A18: Finite stationary tax increase from .189 to .199

Economy	Direct loss	Integrated ETI	Integrated correction
One-type ENOP	0.001728709	0.001615964	0.000112746
Ladder, OTJ off	0.001695048	0.001583953	0.000111095
Ladder, OTJ on	0.001717919	0.001586172	0.000131747

Losses are differences in equilibrium flow welfare between the two tax rates, in common goods units. The integrated columns use the ETI and correction terms in (3.27).

either the thirteen-equation two-type system or its seven-equation one-type limit. It does not impose tax invariance or an analytical participation response. Positive variables use transformed coordinates; feasibility, resources and ranking are checked after solving. Implicit derivatives differentiate the original equilibrium equations. Independently solved nearby-tax equilibria provide central-difference checks.

The baseline equilibrium audit obtains the same respective roots from 72 perturbed starts across six cases, with maximum original equation error 1.78×10^{-15}. The planner audit checks twelve resource/restricted cases and 96 perturbed starts, with maximum equation error 4.45×10^{-15}. These multistart checks provide numerical evidence of regularity around the reported solutions, not global uniqueness or global OTJ planner optimality.

Eight-point Gauss–Legendre quadrature integrates the welfare decomposition in (3.27) over the tax interval. Across the baseline finite-tax comparisons, the largest difference between directly computed welfare loss and the sum of the integrated ETI prediction and correction is 4.55×10^{-16}.

Table A19: Admissible target-preserving latent calibrations

k	R_{on}	R_{off}	$100\Delta R$	H_{on}	H_{off}	$(Z/Y)_{\text{on}}$
0.25	1.073166	1.071480	0.1686	0.165341	0.165052	0.836503
0.50	1.077916	1.071604	0.6312	0.165463	0.164385	0.833432
0.75	1.081983	1.071876	1.0107	0.165567	0.163846	0.830820
1.00	1.085504	1.072201	1.3302	0.165656	0.163397	0.828572
1.50	1.091295	1.072906	1.8389	0.165802	0.162693	0.824901
2.00	1.095862	1.073624	2.2238	0.165916	0.162169	0.822029
3.00	1.102602	1.075014	2.7588	0.166083	0.161458	0.817826
4.00	1.107337	1.076312	3.1025	0.166199	0.161016	0.814899

$k = a_G/a_B$ indexes the OTJ-on calibration. The maintained ON targets, $A/Z = .11$, cash-wage premium and implied ETI are the same in every row. Each OFF counterpart changes only o, retaining the goods units of its ON parent. Its own ETI, wage premium and amenity ratio may change. The .10 point fails worker ranking. The interval [.25, 4] is a studied profile, not an estimated identified set or global bound.

Table A20: Lower ETI with baseline allocations preserved

Value	ε_{on}	R_{on}	R_{off}	ΔR, pp	ΔH, %
0.790323	0.7903	1.085504	1.072201	1.3302	1.3825
0.4	0.4000	1.072013	1.060750	1.1263	1.3844
0.3	0.3000	1.062908	1.053034	0.9874	1.2645
0.2	0.2000	1.044698	1.037632	0.7066	0.9446

Benchmark aggregate targets. Each off economy changes only o within its calibrated pair. These are diagnostic parameter grids, not estimated parameter intervals.

G.2 Latent calibrations and sensitivity tables

Table A19 reports the admissible target-preserving latent calibrations. Tables A20–A23 report the parameter sensitivity exercises. The amenity profile excludes $k = .10$. Near-indifference checks, $\beta = .99999$ and $\nu = 50$ remain admissible; the latter two give $R_{\text{on}} = 1.000219$ and 1.010934, respectively. No tested interior point has a nonpositive correction.

An additional experiment changes only baseline o, using multipliers 0, .25, .5, .75, 1, 1.25, 1.5, and 2. All eight cases pass the feasibility checks; R increases from 1.072201 at zero to 1.094112 at twice baseline exposure. This is a change in productive OTJ opportunities, unlike a target-preserving scale recalibration. Complete allocations and derivatives under

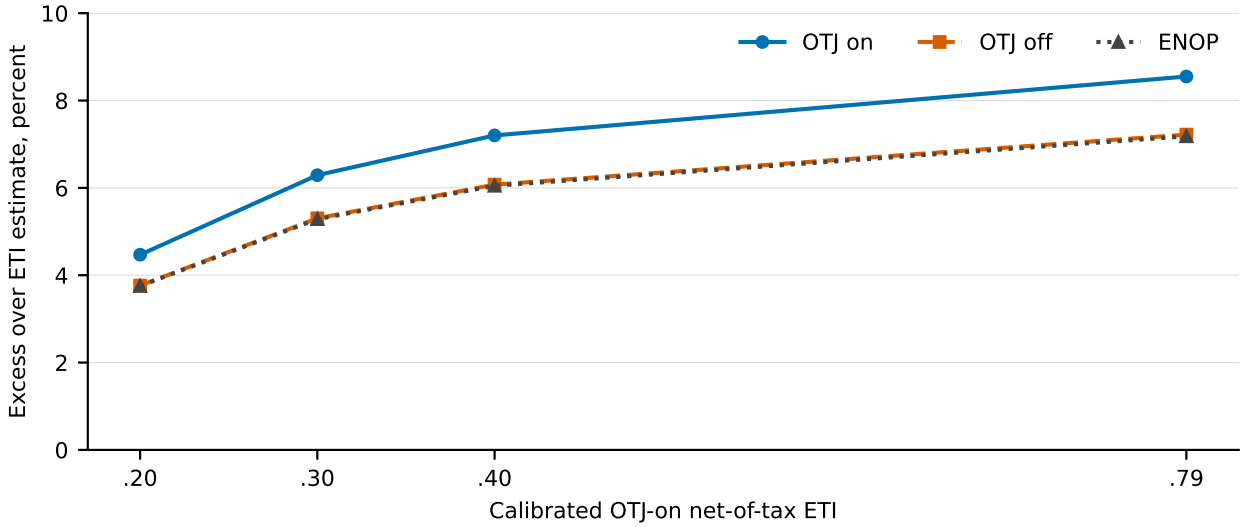


Figure A1: Lower ETI with the OTJ baseline allocation preserved

Only participation curvature and its utility scale change at calibration. The off ETI adjusts endogenously; its value is not forced to equal the horizontal coordinate. ENOP is recalibrated at the same curvature and aggregate targets. The lines use distinct markers and styles as well as color.

Table A21: Discount-factor sensitivity

Value	ε_{on}	R_{on}	R_{off}	ΔR, pp	ΔH, %
0.99	0.7903	1.207395	1.178690	2.8705	2.7490
0.994	0.7903	1.126878	1.107917	1.8961	1.9154
0.996	0.7903	1.085504	1.072201	1.3302	1.3825
0.998	0.7903	1.043254	1.036240	0.7014	0.7509
0.999	0.7903	1.021761	1.018157	0.3604	0.3918
0.9999	0.7903	1.002189	1.001819	0.0370	0.0408

Benchmark aggregate targets. Each off economy changes only o within its calibrated pair. These are diagnostic parameter grids, not estimated parameter intervals.

Table A22: Matching elasticity under Hosios bargaining

Value	ε_{on}	R_{on}	R_{off}	ΔR, pp	ΔH, %
0.4	0.7903	1.069766	1.059007	1.0759	1.1562
0.5	0.7903	1.085504	1.072201	1.3302	1.3825
0.6	0.7903	1.109110	1.091689	1.7421	1.7418

Benchmark aggregate targets. Each off economy changes only o within its calibrated pair. These are diagnostic parameter grids, not estimated parameter intervals.

Table A23: Alternative aggregate amenity shares

Value	ε_{on}	R_{on}	R_{off}	ΔR, pp	ΔH, %
0.05	0.6935	1.093010	1.077927	1.5083	1.5605
0.11	0.7903	1.085504	1.072201	1.3302	1.3825
0.2	0.9355	1.077156	1.065842	1.1314	1.1818

Benchmark aggregate targets. Each off economy changes only o within its calibrated pair. These are diagnostic parameter grids, not estimated parameter intervals.

both aggregate conventions are retained in the machine-readable results.

G.3 A scalar off-economy solution and independent checks

For the repeated two-type no-OTJ comparisons, common bargaining weights permit a scalar solution in $Q = h'/t$:

$$J_i(Q) = \frac{y_i - Q}{d_0(1+b)}, \quad q_i = \frac{\gamma_i}{\beta J_i}, \quad \theta_i = (\phi/q_i)^{1/(1-\xi)}, \quad f_i = \phi\theta_i^{\xi}. \tag{G.1}$$

The scalar equation is $Q = \beta b[(1-\alpha)f_B J_B + \alpha f_G J_G]$. Its right side decreases on the positive-value domain. The implementation checks for a bracketed root in $0 < Q < \min_i y_i$, then recovers

$$\ell = \left(\frac{\zeta}{tQ}\right)^{1/\nu}, \quad s = \frac{1-\ell}{1+f_U/\rho}, \quad n_i = \frac{c_i f_i s}{\rho}, \quad v_i = \theta_i c_i s, \tag{G.2}$$

where $c_B = 1-\alpha, c_G = \alpha$. Amenities are $a_i = \zeta_i^a/(tn_i)$ and cash pay is $w_i = y_i - d_0 J_i - a_i$. Calculating cash pay separately from the root finder exposes negative-pay failures.

The robustness audit includes 292 full-system level and implicit derivative comparisons, including 48 selected bootstrap pairs. Maximum transformed-level disagreement is 1.38×10^{-13} and derivative-metric disagreement is 9.55×10^{-15}. Sixty-four central-difference comparisons at two step sizes have maximum absolute goods-loss error 9.17×10^{-9}. All admitted bootstrap cases satisfy original equations with maximum residual below 6.16×10^{-14} and reproduce their calibration targets within 8.89×10^{-16}.

Numerical tables and headline values are generated from the saved equilibrium results.

The accompanying replication materials document the inputs, equilibrium and planner calculations, calibration restrictions and derivative checks.